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A METHODOLOGY FOR SELECTING SMALL-ARMS ROUNDS TO MEET MILITARY REQUIREMENTS

CENTER FOR NAVAL ANALYSES
1401 Wilson Boulevard
Arlington, Virginia 22209

Marine Corps Operations Analysis Group

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This paper develops a methodology for determining the characteristics of candidate small-arms rounds that satisfy a given set of military requirements. Ballistics-related information required for the methodology (and of general interest to the small-arms field) is also presented.

The intended user of the methodology is the military project officer or analyst rather than the munitions developer; consequently, the approach taken is primarily operational rather than technical.

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INTRODUCTION

Since World War II, there have been numerous efforts to develop small-arms systems that would increase the effectiveness of the infantryman. For some time, it appeared that unconventional systems would provide the greatest improvement. Unfortunately, their development has not lived up to early expectations. It appears likely that the next generation of small arms will be of conventional design and fire a ball round, though the cartridge case may not be brass. In fact, more recent research has been concerned with conventional systems. In general, this research has not been well organized and has suffered from a lack of methodology and suitable measures of effectiveness. General field tests of the M16, 5.56-mm. and M14, 7.62-mm. rounds have been carried out without well-defined and documented purposes. Frequently, results of such tests were interpreted as comparisons of the two calibers rather than of the two weapon systems or of the particular rounds employed.

The primary purpose of this research contribution is to develop a methodology that allows determination of the candidate small-arms rounds that satisfy a given set of military requirements. In the event that no candidate round can satisfy a particular set of requirements, the methodology presented provides insights that will help define a realistic set. Thus, while formulating requirements is primarily a responsibility of the military services, this analysis and methodology can assist with this formulation. The approach taken in this paper is primarily operational rather than technical, since the intended user is the military project officer or analyst rather than the munitions developer or arms manufacturer.

Examination of requirements, studies, tests, position papers, and comments on combat experience reveals several pervasive small-arms issues. These are: the roles of small arms in modern warfare, the "family of weapons concept," and, finally, the choices of caliber. While these issues are highly interdependent and not at all simple to resolve, some discussion of them is desirable in order to indicate their relationship to our methodology.

Four weapon types are considered in this paper: the individual rifle (IR), the automatic rifle (AR), the light machine gun (LMG), and the medium machine gun (MMG). Traditionally, the rifle has been a point-fire weapon and machine guns have been area-fire weapons, while the automatic rifle's role has been intermediate. Rifle targets are primarily personnel, while heavier small arms, in particular the medium machine gun, have frequently been employed against "thin-skinned" vehicles. Thus, both the targets and weapon fire roles change as we progress from the rifle to the medium machine gun. Also, while people with infantry experience often have rather firm ideas about requirements, there is little firm basis for making quantitative statements about targets and weapon roles. Post World War II trends appear to favor an "assault" rifle that can be converted to an automatic rifle by a selector switch and sometimes substituting a heavier barrel. The M60 machine gun fills the role of the MMG, but the

LMG is absent from current tables of equipment. At the same time, various weapon manufacturers continue to offer automatic weapons (frequently in 5.56mm) that could be classified as light machine guns.

During the last decade, the "family of weapons concept" has received considerable attention. While an adequate definition of this concept is difficult to find, it includes weapon types from the IR to the LMG and perhaps the MMG. The purpose of the concept is to maximize commonality. The family must fire the same round, have a high proportion of common parts, employ common assemblies, and, in its most advanced form, permit battlefield reconfiguration from one weapon type to another. The United States employed a common round in World War II. The 30-06 round was fired in the rifle (Garand), the automatic rifle (BAR), and the 30-caliber machine guns (LMG and MMG). Therefore, the common round capability is not new. The capability to reconfigure weapon types with a minimum change of parts is what makes the "family of weapons concept" new. While this concept is enticing to the military, it probably has received the most attention from civilian researchers and weapons manufacturers. The benefits of being able to "convert everything to rapid-fire machine guns" in case of a heavy, enemy infantry attack have been mentioned many times. The logistical advantages of a common round and fewer parts have also been pointed out on numerous occasions. The fact that machine guns consume much more ammunition than rifles (ammunition that small infantry units would have to transport) has received little attention. The fact that machine guns use linked ammunition while rifles employ box magazines is usually overlooked -- though this difference negates many advantages of a common round.

While a common round is a basic requirement of the "family of weapons concept," it is desirable even if the family concept is abandoned. Then, the basic question becomes: Will a common round suffice (meet requirements) for all weapons from the individual rifle through the medium machine gun, or are at least two different rounds required? The resolution of this question hinges on performance requirements. Development of a methodology for solving problems relating to round selection is prerequisite to an orderly development of small-arms systems satisfying a set of requirements.

The methodology presented in this paper can be used to determine candidate small-arms rounds that satisfy a given set of military requirements. Once the candidate rounds satisfying the formulated set of requirements have been determined, rounds can be fabricated and range tested for performance. Much of this testing can be carried out before the candidate weapons have been developed. For example, muzzle velocity, down-range energy, and penetration capability can be tested in single-shot rifles without waiting for development of the candidate weapons that will employ these rounds. Evaluation of the candidate weapon systems is outside the scope of this paper. It involves field testing of those systems that analysis and small-arms knowledge indicate to be the most promising.

While developing the methodology given in this research contribution, detailed technical information relating to interior, exterior, and terminal ballistics was collected and analyzed. This material is contained in the four appendixes and much of it is used in developing the methodology. Appendix A deals with weapon round parameter relations, B with round design and rifling twist, C with impulse, recoil and accuracy, and D with down-range performance.

METHODOLOGY

Bullet calibers between .224 and .308 inches represent the practical range of calibers to be considered for the "family of weapons concept." Engineering requirements limit the practical range of bullet weights for any given caliber.[1] Bullet weight (w), muzzle velocity (v_o), and peak chamber pressure ($\hat{p}$)[2] are used to describe candidate rounds.

To graphically display this information, the muzzle velocities and the bullet weights associated with a particular peak chamber pressure are plotted. The candidate rounds are the rounds lying interior to the space bounded by the maximum and minimum calibers and the maximum and minimum practical bullet weights. This space or region is called the "peak chamber pressure space" and includes all candidate rounds at a particular peak chamber pressure.

Next, the bounds imposed by military requirements are plotted, which determine a subspace of the peak chamber pressure space. This subspace is called the solution space and represents all rounds satisfying the military requirements as shown in figure 1. The formulas and numerical values required to construct solution spaces are contained in the appendixes.

Appendix A develops formulas for homologous rounds and applies these formulas to determine numerical values for five bullet-weight categories: Typical Heavy (TH), Medium/Heavy (M/H), Typical Medium (TM), Medium/Light (M/L), and Typical Light (TL). The five case-volume categories are: Typical Large (TL), Medium/Large (M/L), Typical Medium (TM), Medium/Small (M/S), and Typical Small (TS). The extremes of these categories represent the practical engineering limits for each caliber. The bullet-weight and case-volume categories span those common in military and commercial use. The range of calibers includes all those commonly employed in the small-arms role. Thus, all rounds (caliber, bullet weight, and case volume) that hold promise as candidates to meet typical small-arms military requirements are included in the solution space. The numerical values of these bullet and case categories are calculated for each of five calibers denoted by bullet diameter d = .224, .243, .264, .284, and .308 inches.

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[1] A very light-short bullet will not have enough bearing surface to properly engage the rifling of the bore and a very long-heavy bullet is difficult to stabilize.

[2] Mean effective pressure ($\tilde{p}$) would be a better descriptor, but peak chamber pressure is more commonly used in small-arms literature. (See appendix A.)

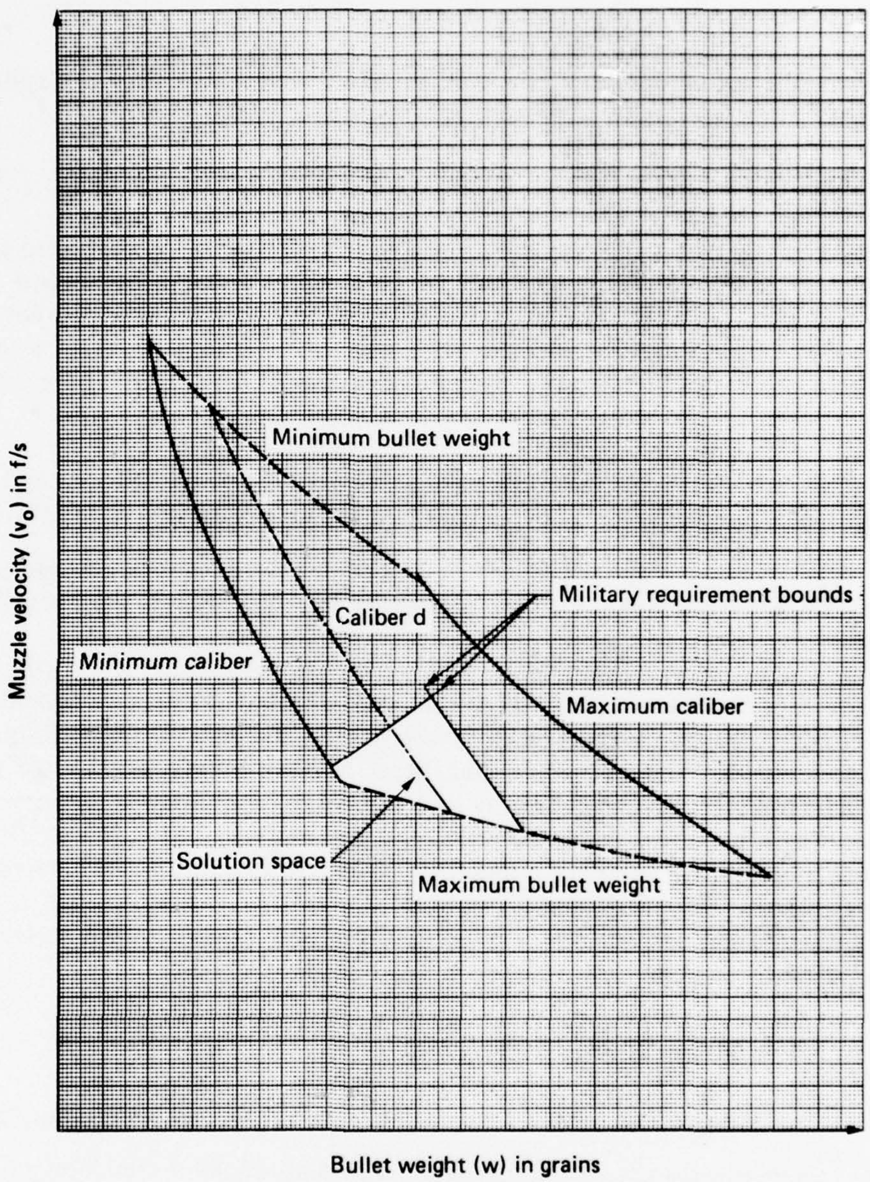

FIG. 1: PEAK CHAMBER PRESSURE SPACE SHOWING THE SOLUTION SPACE ($\hat{p}$ IS CONSTANT)

Thus, in figure 1, the peak chamber pressure space is bounded above by minimum bullet weights equal to the TL category and below by maximum bullet weights equal to the TH category. The left caliber bound is the minimum bullet diameter (.224 inches), while the right caliber bound is the maximum (.308 inches).

Requirements statements and how they provide bounds to define the solution space within the peak chamber pressure space are discussed next.

REQUIREMENTS STATEMENTS

Military requirements for new small-arms weapons vary according to the source. In general, requirements statements also vary, ranging from rather broad descriptive statements to more specific quantitative statements. Furthermore, the set of small-arms parameters important for analytic purposes may not be the same as the set critical for operational purposes. However, the two sets should have a high percentage of common elements.

The five parameters that are most useful for analytic purposes are caliber (d), bullet weight (w), muzzle velocity (v_o), mean effective pressure ($\tilde{p}$), and ballistic coefficient ($\overline{C}$). These parameters are not independent. Additional parameters important for analysis are: peak chamber pressure ($\hat{p}$), free recoil (R), recoil impulse (J), and bullet energy (E).

Requirements that are critical from the point of view of military operations are maximum effective range (MER), weapon weight (W), accuracy, and reliability. This paper will not consider accuracy or reliability. Military arms testing has already devoted considerable attention to accuracy -- in its various forms. Furthermore, there is no reason why a properly designed round, in any caliber from .224 to .308, should not be accurate within its range of capability. Also, there is no established reason why a properly designed weapon, firing a round in this caliber range, should not be reliable. However, good weapon design, while important for both accuracy and reliability, appears to be more difficult to achieve than good round design.

In selecting a set of requirements to illustrate the methodology, two criteria were employed:

- The requirements should be reasonable from a military point of view.
- They should illustrate the basic elements of the methodology.

The latter criterion is our reason for increasing the number and specificity of the requirements beyond what is usually found in military statements.

Table 1 gives various parameter values for the M16 and M14 rifles.

TABLE 1

PARAMETER VALUES FOR M16 AND M14 RIFLES

Rifle	M16	M14
Caliber, d	.224 in.	.308 in.
Rifle weight (loaded), W	7.6 lb.	10.7 lb.
Bullet weight, w	55 grains	147 grains
Muzzle velocity, v_o	3,250 fps	2,800 fps
Muzzle energy, E_o	1,290 ft-lb.	2,560 ft-lb.
Peak chamber pressure, $\hat{p}$	52,000 psi	50,000 psi
Mean effective pressure, $\tilde{p}$	17,860 psi	18,740 psi
Case volume, V_c	.112 cu.in.	.198 cu. in.
Free recoil (loaded), R	3.0 ft-lb.	10.2 ft-lb.
Recoil impulse, J	1.2 lb-sec.	2.6 lb-sec.
Bore length, ℓ	18.4 in.	20.3 in.

Since their calibers (.224 and .308) are the minimum and maximum calibers considered in this paper, their parameter values serve to illustrate a range of reasonable military requirements. The set of military requirements selected to illustrate the methodology is given in table 2.

Some discussion of each of these requirements is helpful in relating them to the two selection criteria stated above.

Maximum Effective Range (MER)

There is no generally accepted definition of the maximum effective range for small arms. Definitions such as "maximum effective range (MER) is the maximum range at which economical use of the weapon may be expected" (page A-23 of reference 1) are not suitable for analysis purposes. What is required is a definition that specifies the maximum range at which a given level of casualty producing capability is obtained.

It is rather widely accepted that casualty producing capability is related in some way to bullet weight and velocity. Kinetic energy of a bullet, and the ability of a bullet to transfer energy to a target, constitute more refined versions of the bullet-weight velocity concept of lethality. Kinetic energy is easy to determine but energy transfer capability is not. Various measures of effectiveness based on penetration have been proposed, such as the ability of a bullet to penetrate a helmet or a given thickness of pine boards. The "Ballistic Research Laboratories (BRL) three-halves incapacitation probability formula" (page D-181 of reference 1) involves velocity to the three-halves power and appears

to be a step towards developing a generally acceptable formula. However, it has not been validated to the extent where general acceptance has followed.

TABLE 2

EXAMPLE SMALL-ARMS REQUIREMENTS

(1) The maximum effective ranges (MER, meters) required for the four basic weapon types are: IR, 500; AR, 500; LMG, 700; MMG, 1,000.

(2) The rifle weight (W) is not to exceed 9 pounds.

(3) The free recoil (R) for a rifle of weight 9 pounds is not to exceed 9 foot-pounds.

(4) The muzzle velocity of (v_o) of a round fired in a bore of length (ℓ) of 22 inches is not to exceed 3,200 feet per second.

(5) The peak chamber pressure ($\hat{p}$) is to be 50,000 pounds per square inch.

(6) The height of the trajectory (H_r) is not to exceed 12 inches over 300 meters (range r).

(7) It is desirable that a common round be employed in all four weapon types. However, two separate rounds will be considered.

For the methodology of this paper, all that is required is that the effectiveness measure be expressible in terms of bullet weight and velocity. Therefore, we shall determine the energy required for a bullet to be effective as a function of caliber. (See formula D-15 of appendix D.) This formula is based on helmet penetration data from BRL and gives the kinetic energy required for helmet penetration with sufficient residual energy (after penetration) to produce a "serious wound." Thus, the energy required at maximum effective range (MER-E) can then be calculated (by formula D-15) as a function of caliber. The MER-E in foot-pounds calculated for each of the five basic calibers is given in table 3.

The maximum effective range requirements of table 2 are based on the following logic. The maximum effective range for the IR and AR are equal because they may be identical weapons and differ merely by the position of a selector switch setting. Most personnel targets are engaged at ranges under 300 meters, however, 500 meters was selected to provide reserve effectiveness beyond 300 meters. The LMG has a somewhat different role from the IR and AR, and its maximum effective range was taken as 200 meters greater than that of the rifles. It seems desirable to require that the

MMG have a greater maximum effective range than the LMG, so its MER was selected as 1,000 meters. These effective range requirements may not agree closely with MER requirements specified at various times by the U.S. military, but they are sufficiently reasonable for demonstrating the methodology.

TABLE 3

ENERGY REQUIRED AT MER

Cal. (d)	MER-E (ft-lbs.)
.224	305
.243	333
.264	364
.284	396
.308	435

Rifle Weight (W)

The rifle weight is limited to 9 pounds, which is roughly the average of the loaded weights of the M16 and M14 rifles.

Free Recoil (R)

The free recoil (recoil energy) of the rifle is limited to 9 foot-pounds. This is somewhat less than the recoil of the M14 but considerably more than that of the M16. It is necessary to translate rifle weight (W) and free recoil (R) into some form which permits plotting a "rifle weight and free recoil" bound on the peak chamber pressure space conceptualized in figure 1. Fortunately, this translation evolves quite logically. First of all, the only way to estimate (in theory) the free recoil for a given rifle weight (W), bullet weight (w), and muzzle velocity (v_o) is to use recoil impulse (J). The relation of free recoil and rifle weight to recoil impulse is given by formula C-3 of appendix C:

$$J = \sqrt{\frac{2\,WR}{g}} \;.$$

Recoil impulse is estimated from muzzle velocity and bullet weight by formula C-4:

$$J = w\left[-2.48\,(10^{-10})\,(v_o)^2 + 4.45\,(10^{-6})v_o + \frac{.014v_o - 10.12}{6823 - v_o}\right].$$

From formula C-3, when W and R are equal to their maximum values, J equals its maximum value. Hence, if the maximum rifle weight is 9 pounds and the maximum free recoil is 9 foot-pounds (see table 2), the maximum recoil impulse is 2.24 pound-seconds. Thus, the upper bounds on rifle weight and free recoil can be replaced by the implied upper bound on recoil impulse (J). J (being a function of bullet weight (w) and muzzle velocity (v_o) in formula C-4) can then be plotted on the $\hat{p}$-space conceptualized in figure 1.

Muzzle Velocity (v_o) and Bore Length (ℓ)

The maximum muzzle velocity is limited to 3,200 feet per second from a bore 22 inches long.[1] Muzzle velocities in excess of 3,300-3,400 feet per second are thought to result in excessive barrel wear,[2] especially at the high rates and volumes of fire associated with military use.

Peak Chamber Pressure ($\hat{p}$)

The peak chamber pressure selected is 50,000 psi because this is considered nominal. Peak chamber pressures much in excess of 52,000 psi begin to introduce requirements for heavier receivers and can cause cartridge case extraction problems.

Height of Trajectory (H_r)

The height of trajectory is limited to 12 inches. With standard battlesight zeroing at 300 meters, the bullet will rise a maximum of 12 inches above the line of sight for targets closer than 300 meters. Reducing the value of this requirement reduces the maximum distance a bullet would strike above the point of aim for targets closer than 300 meters.

Round Options

Requirement (7) states the desirability of a common round with consideration being given to two separate rounds for the four weapon types. The two-round option leaves the question of the exact commonality open. Therefore, solution spaces will be examined at 50,000 psi peak chamber pressure for the following three options:

---

[1]This bore length for all weapons was selected to represent the bore length of the LMG and MMG. Shorter bore lengths on the IR and AR would reduce the muzzle velocities approximately 30 fps per inch.

[2]Excessive barrel wear is probably caused by high chamber temperatures associated with high pressures and resulting high muzzle velocities.

Option I. A common round for all four weapon types (IR, AR, LMG, MMG),

Option II. Two rounds, one for the IR and AR and a second round for the LMG and MMG, and

Option III. Two rounds, one for the IR, AR, and LMG and a second round for the MMG.

Solution spaces for all three of these options will be developed for a 50,000 psi peak chamber pressure (requirement (5)). Option I (common round) solution spaces will also be developed for 52,000 and 48,000 psi peak chamber pressures to demonstrate the effect of peak pressure on the common round solution space. The Option I solution spaces for these three peak chamber pressures will then be combined to show a common round solution space over the peak chamber pressure range of 48,000 to 52,000 psi.

The 50,000 psi Peak Chamber Pressure Space

The first step in developing a solution space is to construct the 50,000 psi peak chamber pressure space to meet requirement (5) of table 2. This requires knowing the bullet weights and muzzle velocities for all five bullet categories of each of the five basic calibers so that these values can be plotted. The bullet weight and muzzle velocity combinations are calculated in appendix A (see table A-7) and summarized in table 4. Requirement (1) of table 2 makes it necessary to know the energies of each bullet at ranges 500, 700, and 1,000 meters. These are calculated in appendix D (see table D-4) and also shown in table 4. Table 4, therefore, contains all the data required to construct the 50,000 psi peak chamber pressure space and determine the MER-E and J bounds leading to solution spaces meeting the requirements of table 2.

Plotting the bullet weights and muzzle velocities (w, v_o) for each basic caliber (d), and drawing the dashed TL and TH bullet category bounds, defines the 50,000 psi peak chamber pressure space shown in figure 2.

Curves are shown for all five basic calibers, two of which (.244, .308) form the left and right bounds of this space.[1]

Option I Solution Space

The Option I solution space must satisfy the maximum effective range requirement of 1,000 meters since the common round is fired in all four weapon types. To construct the MER-E bound at 1,000 meters, the $E_{1,000}$ values of table 4 are plotted as a function

[1] The caliber for any point (w, v_o) of the space can be calculated from formula A-55 using $\tilde{p}$ = 20,390 psi from table A-9. The associated case volumes (v_c) can be estimated from figure A-8.

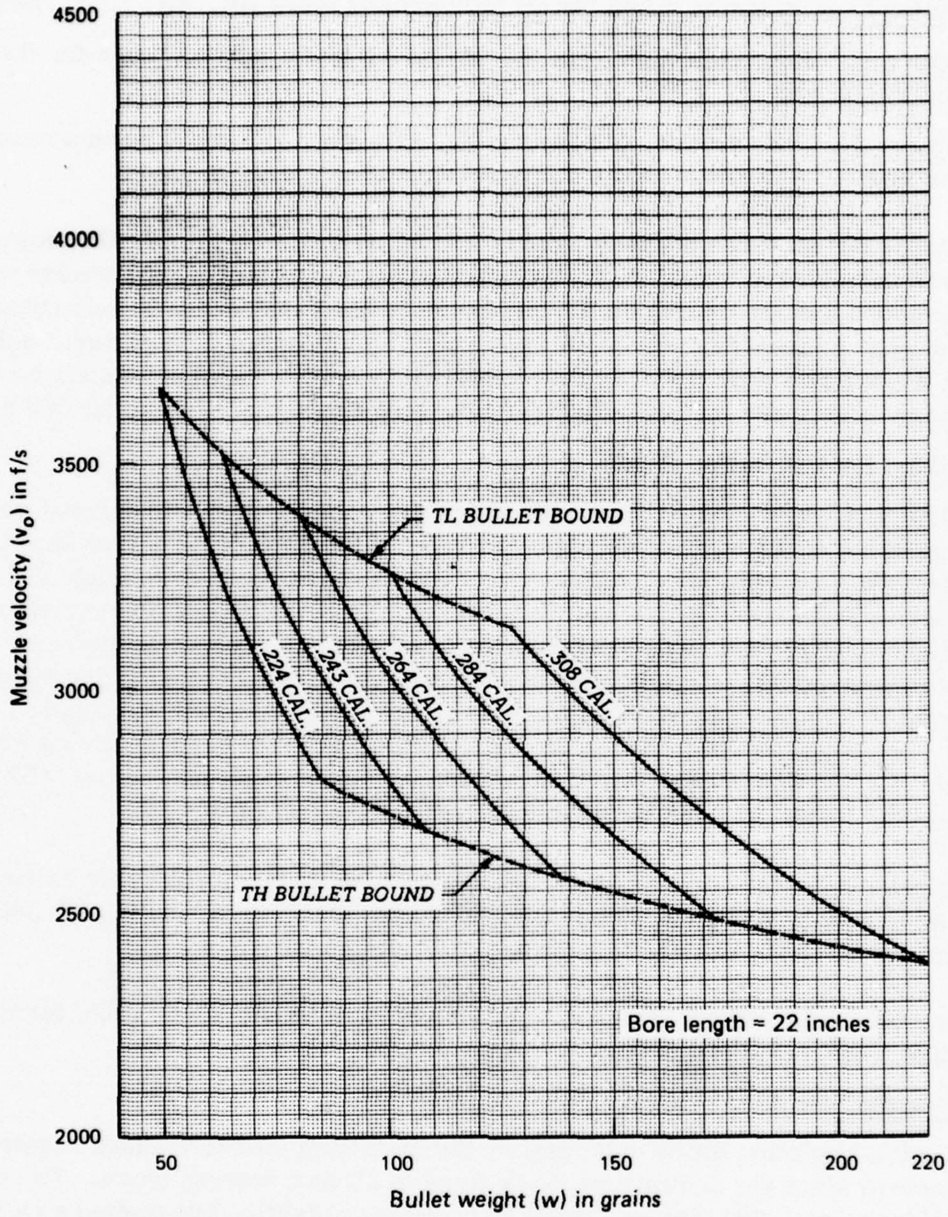

FIG. 2: 50,000 psi PEAK CHAMBER PRESSURE SPACE

TABLE 4

VELOCITY AND ENERGY DATA FOR: PEAK CHAMBER PRESSURE-50,000 psi, BORE LENGTH-22 INCHES, CASE CATEGORY-TM

d (inches)	.224	.243	.264	.284	.308
MER-E (ft-lb.)	305	333	364	396	435
TH Bullet Category					
w (grains)	84.40	107.7	138.2	172.0	219.4
J (lb-sec)	1.50	1.82	2.21	2.62	3.18
v_o (fps)	2804	2692	2582	2490	2391
E_{500} (ft-lb.)	607	759	951	1158	1430
E_{700} (ft-lb.)	415	535	689	858	1085
E_{1000} (ft-lb.)	246	327	435	558	728
M/H Bullet Category					
w	75.56	96.47	123.7	154.0	196.4
J	1.44	1.75	2.12	2.52	3.05
v_o	2962	2844	2729	2631	2527
E_{500}	552	697	878	1072	1335
E_{700}	361	472	613	767	980
E_{1000}	206	277	371	477	629
TM Bullet Category					
w	66.73	85.19	109.2	136.0	173.5
J	1.39	1.68	2.03	2.41	2.92
v_o	3153	3027	2904	2800	2689
E_{500}	493	626	795	977	1224
E_{700}	305	402	529	669	864
E_{1000}	169	227	306	397	529
M/L Bullet Category					
w	57.90	73.92	94.78	118.0	150.5
J	1.34	1.61	1.94	2.30	2.78
v_o	3385	3250	3118	3006	2887
E_{500}	424	546	700	867	1093
E_{700}	244	328	438	561	731
E_{1000}	132	179	243	317	425
TL Bullet Category					
w	49.07	62.64	80.33	100.0	127.6
J	1.28	1.54	1.85	2.19	2.64
v_o	3676	3530	3386	3265	3135
E_{500}	345	452	589	736	942
E_{700}	180	248	338	440	586
E_{1000}	98	134	183	240	325
Case volume, V_c (cu. in.)	.1196	.1470	.1661	.1922	.2261

of bullet weight for each of the five basic calibers. Next, the MER-E values of table 3 are located on each caliber curve and the resulting points are connected by a smooth curve as shown in figure 3.

Note that the .224 and .243 caliber bullets do not possess sufficient energy at 1,000 meters to meet the MER-E requirements of table 3. Next, the bullet weights for each caliber curve intersecting the MER-E bound curve for 1,000 meters are read and plotted on the caliber curves of figure 2. Connecting these points results in a curve. This curve is the MER-E bound at 1,000 meters.

From the data of table 4, the recoil impulse (J) values are plotted against the bullet weights for each caliber as shown in figure 4. Then, a horizontal line is drawn at 2.24 pound-seconds.

Only the .284 caliber intersects the 2.24 pound-seconds line. Calibers .264, .243, and a .224 lie entirely below the 2.24 pound-seconds line, while the .308 caliber lies entirely above. Note that in addition to intersecting the .284 caliber curve, the recoil-impulse bound intersects the TL and TH bounds. The bullet weights for each of these intersections are read and located on the .284 caliber, the TL, and TH bullet bound curves of figure 2. (Additional points can be estimated from figures C-2.1 through C-2.6 of appendix C.) A smooth curve is drawn through the points located. This curve is the J bound. The velocity bound (v_o = 3,200 fps) is plotted as a horizontal line in figure 2 and called the v_o bound. The v_o, MER-E, and J bounds are shown in figure 5 plotted over the 50,000 psi peak chamber pressure space. Table D-2 shows that all rounds for all calibers satisfy the height of trajectory (H_r) requirement of 12 inches or less over 300 meters. Hence, the curve for this bound is not shown.

The Option I solution space lies below the curve for the MER-E bound because this is the region of relatively large calibers and heavy bullets capable of meeting the energy requirement at the 1,000-meter range. The solution space also lies to the left of the J (recoil impulse) bound because this region consists of bullet weight and muzzle velocity combinations that do not exceed the impulse constraint ($J \leq 2.24$ lb.-sec.). The solution space also lies above the TH bullet bound because it is the bottom of the 50,000 psi peak chamber pressure space. Thus, the Option I (common round) solution space is a small triangular-shaped region bounded by the MER-E at 1,000 meters, the J bound at 2.24 pound-seconds, and the TH bullet category bound. Of the five basic calibers only the .264 caliber provides common round solutions.[1]

---

[1] The caliber (d) for any point (w, v_o) in the solution space can be calculated from formula A-55 (with ℓ = 22 inches) using $\tilde{p}$ = 20,390 psi from table A-9 and the associated case volumes estimated from figure A-8.

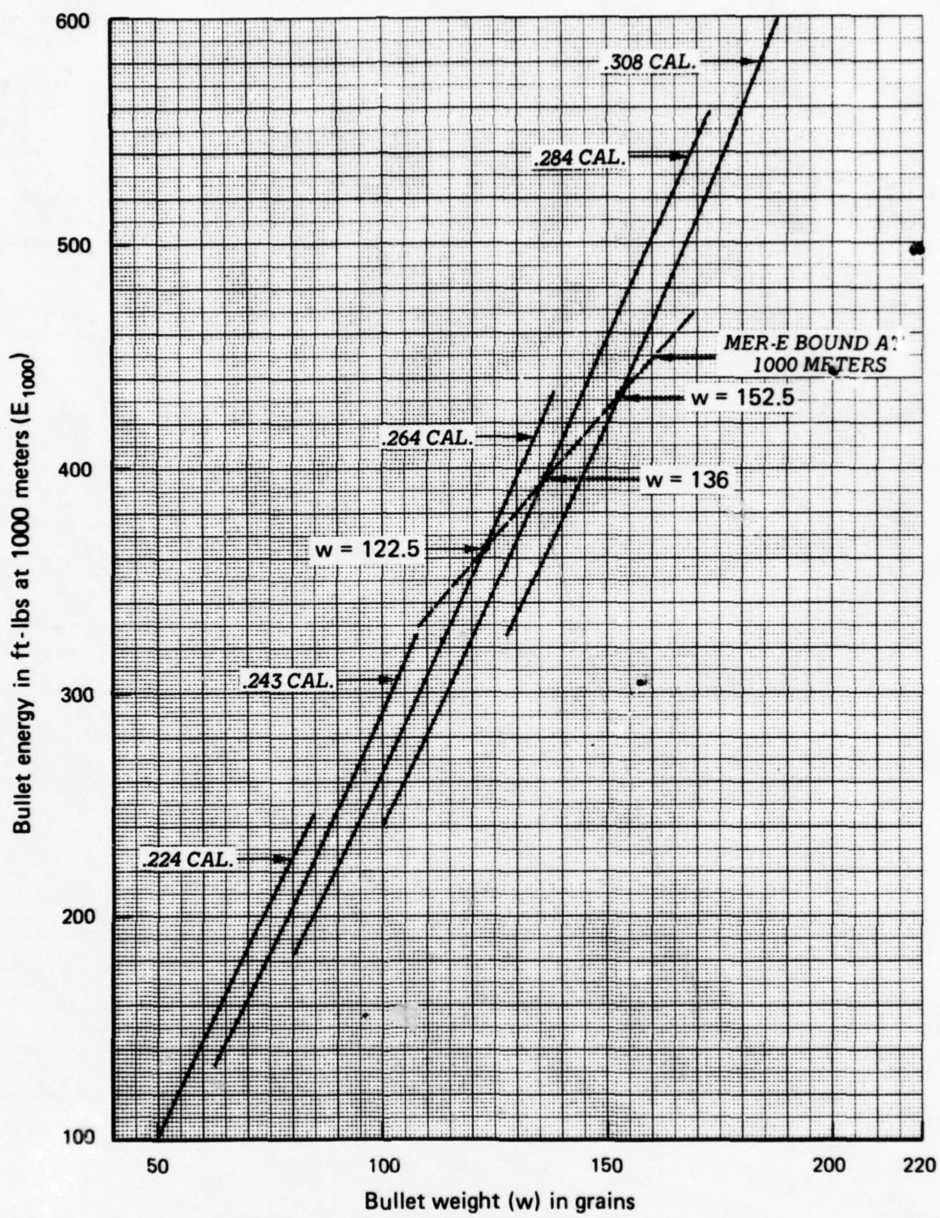

FIG. 3: BULLET ENERGY AT 1000 METERS (E_{1000}), $\hat{p}$ = 50,000 psi

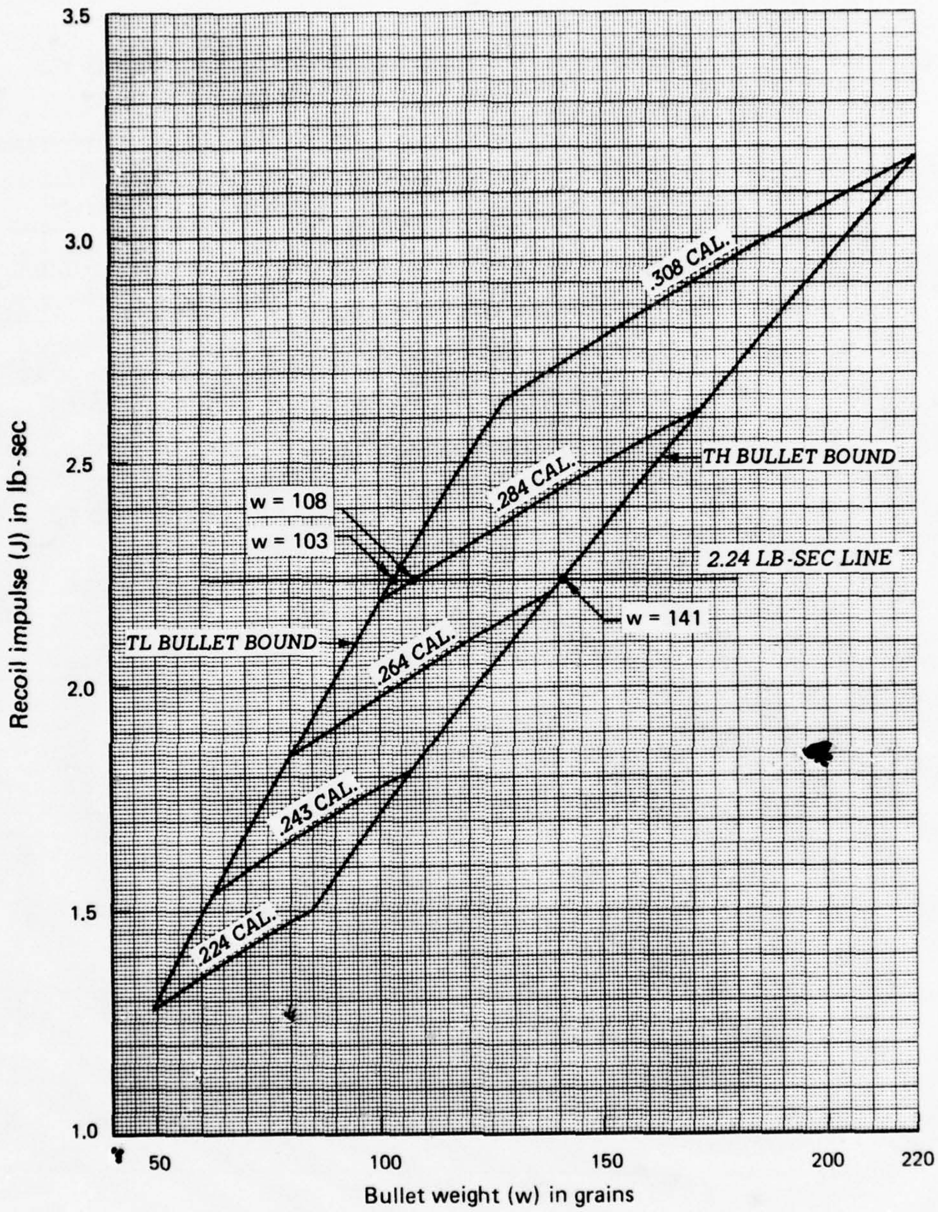

FIG. 4: RECOIL IMPULSE (J), $\hat{p}$ = 50,000 psi

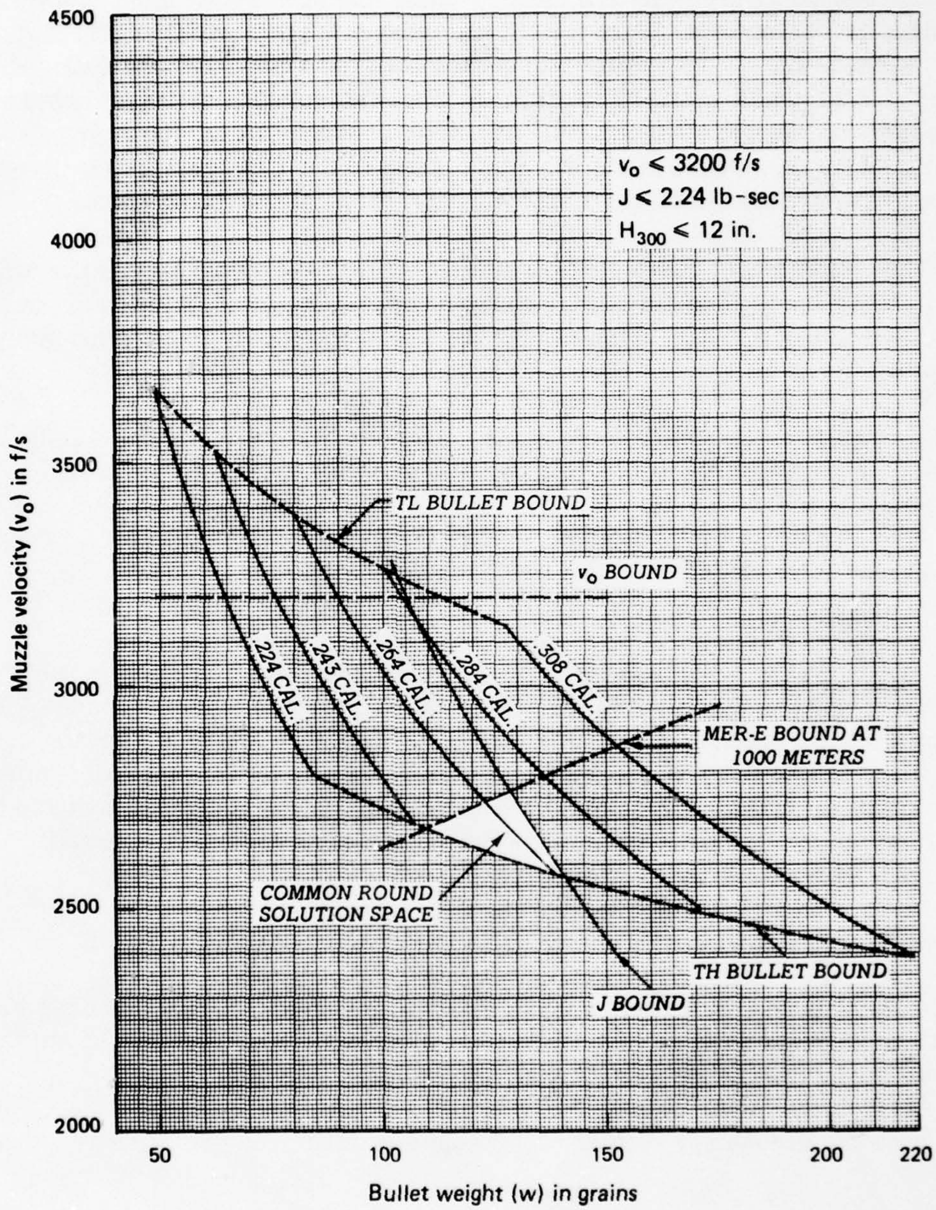

FIG. 5: OPTION I – SOLUTION SPACE, $\hat{p}$ = 50,000 psi

Option II Solution Space

This solution space is generated for two rounds; the first is fired in the IR and AR, and the second in the LMG and MMG. Thus, the first round must meet the MER-E requirement at 500 meters, while the second must meet this requirement at 1,000 meters. Because the J constraint refers to the IR and AR, only the first round must meet the $J \leq 2.24$ pound-seconds condition. Since the solution space involves two rounds, it will be developed as two subspaces. The first subspace is for the round fired in the IR and AR, while the second is for the round fired in the LMG and MMG. These two subspaces overlap in the common round solution space shown in figure 5.

The MER-E bound at 500 meters is constructed in the same way as the MER-E bound at 1,000 meters for Option I. However, all calibers and bullet weights of table 4 exceed the MER-E requirements at 500 meters so the MER-E curve for 500 meters is not plotted.

The J bound for Option II is the same as for Option I because impulse is calculated from bullet weight (w) and muzzle velocity (v_o) and is not affected by caliber or down-range parameter values.

The Option II solution subspaces are shown in figure 6.

Option III Solution Space

This solution space is generated for two rounds, the first is fired in the IR, AR, and LMG, and the second in the MMG. Thus, the first round must meet the MER-E requirement at 700 meters, while the second must meet this requirement at 1,000 meters. Only the first round must meet the $J \leq 2.24$ pound-seconds requirement. The development of the solution space is the same as for Option II, except that the MER-E bound for 700 meters is involved in the solution subspace for the first round. The Option III solution subspaces are shown in figure 7.

THE 52,000 psi PEAK CHAMBER PRESSURE-OPTION I SOLUTION SPACE

Next, we examine the common round solution space if the peak chamber pressure (requirement (5) of table 2) is increased to 52,000 psi.[1] The result is to shift the $\hat{p}$ space upward and the Option I solution space up and to the left.

The velocity and energy data at 52,000 psi peak chamber pressure are shown in table 5.

[1] This is a substantial increase in pressure involving the use of cases whose volumes are slightly larger than M/L as can be seen by plotting the case volume values from table 5 on figure A-8. The result is to increase the muzzle velocities.

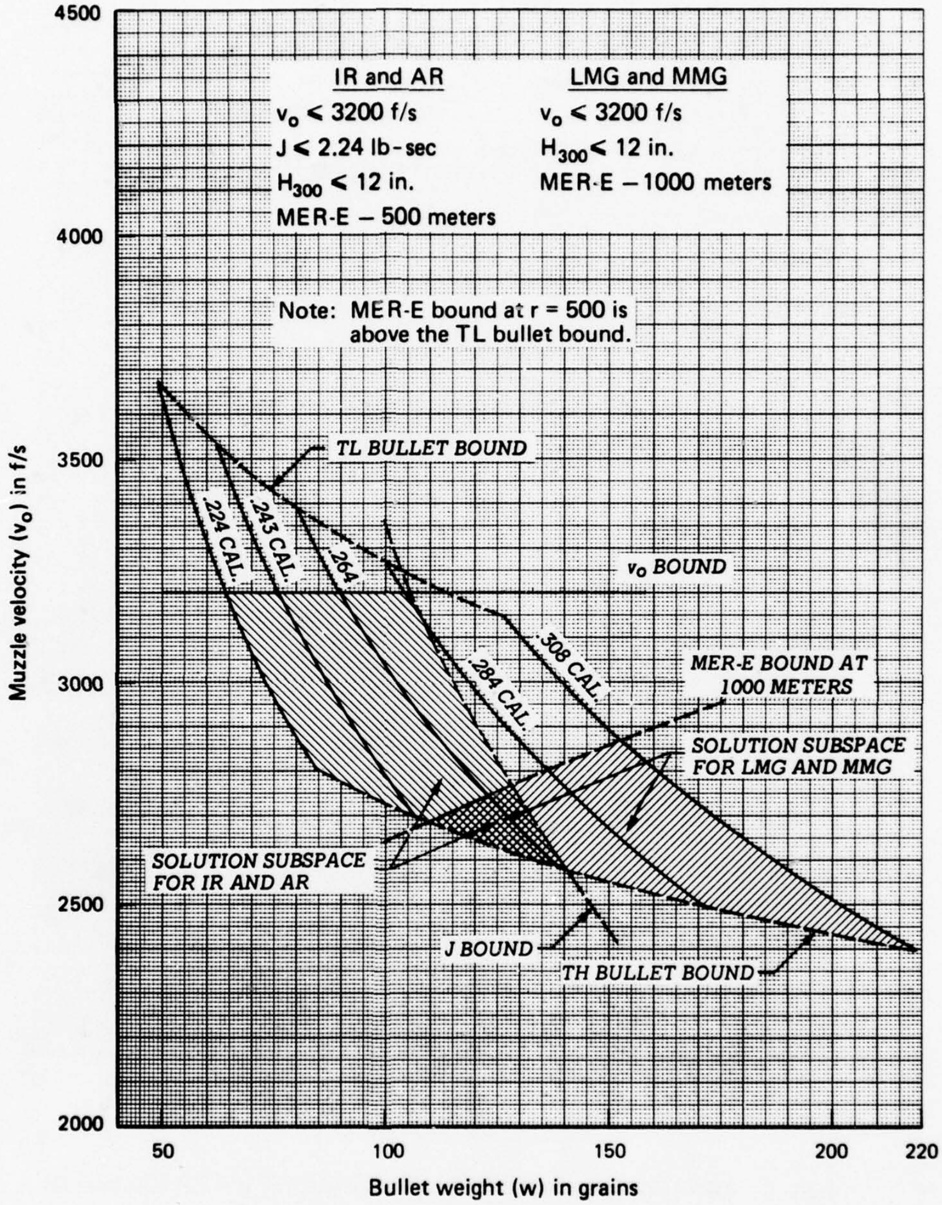

FIG. 6: OPTION II – SOLUTION SUBSPACES, $\hat{p}$ = 50,000 psi

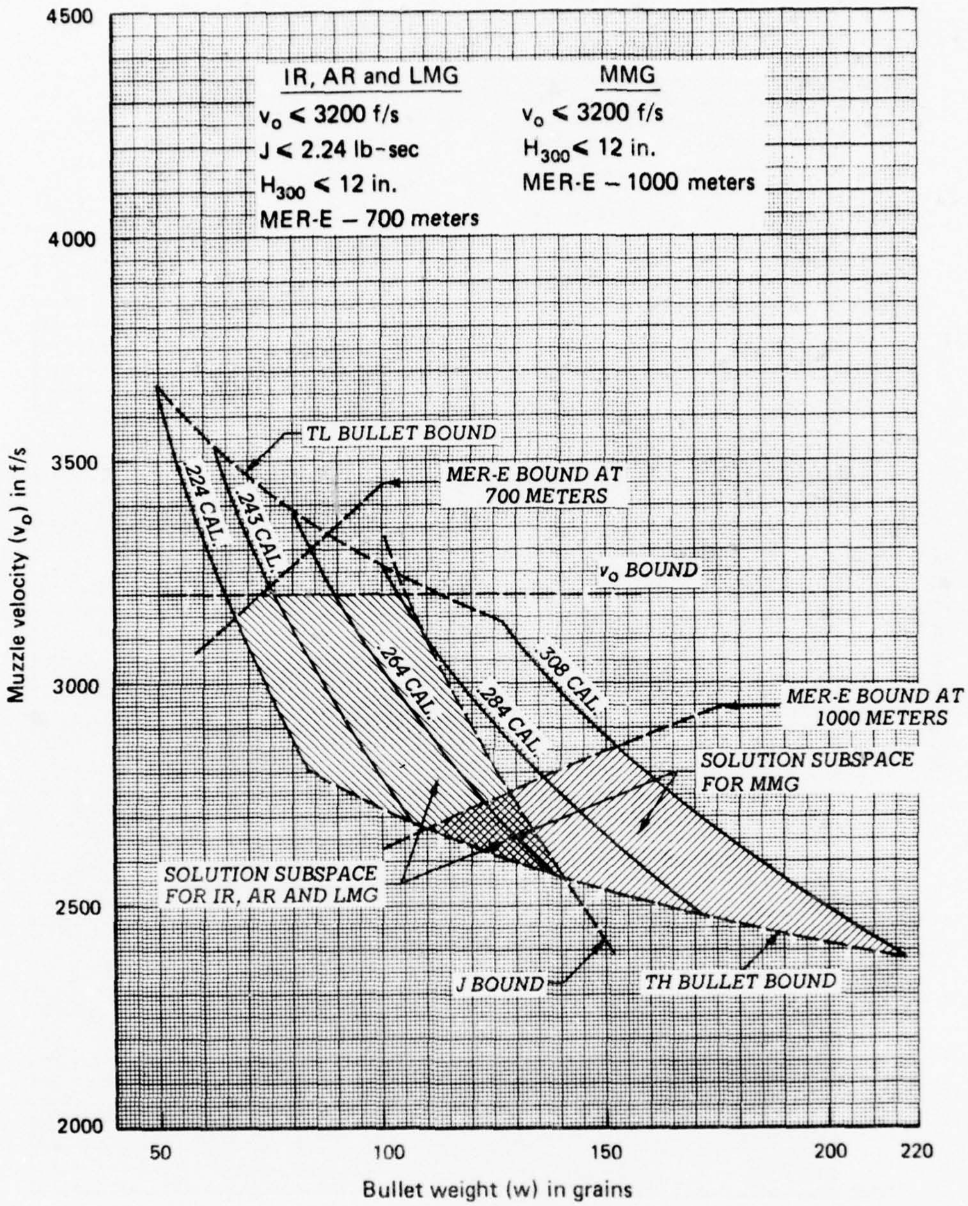

FIG. 7: OPTION III – SOLUTION SUBSPACES, $\hat{p}$ = 50,000 psi

TABLE 5

VELOCITY AND ENERGY DATA FOR: PEAK CHAMBER PRESSURE-52,000 psi
BORE LENGTH-22 INCHES CASE CATEGORY-M/L+

d (inches)	.224	.243	.264	.284	.308
MER-E (ft-lb.)	305	333	364	396	435
TH Bullet Category					
w (grains)	84.40	107.7	138.2	172.0	219.4
J (lb-sec.)	1.66	2.01	2.44	2.90	3.51
v_o (fps)	3031	2910	2792	2692	2584
E_{1000} (ft-lb.)	285	381	512	654	852
M/H Bullet Category					
w	75.57	96.47	123.7	154.0	196.4
J	1.61	1.94	2.35	2.79	3.37
v_o	3203	3075	2950	2845	2731
E_{1000}	236	320	432	558	737
TM Bullet Category					
w	66.73	85.19	109.2	136.0	173.5
J	1.56	1.87	2.26	2.68	3.23
v_o	3408	3272	3140	3027	2906
E_{1000}	191	259	352	461	616
M/L Bullet Category					
w	57.90	73.92	94.78	118.0	150.5
J	1.50	1.80	2.17	2.57	3.09
v_o	3659	3513	3371	3250	3120
E_{1000}	147	201	275	362	489
TL Bullet Category					
w	49.07	62.64	80.33	100.0	127.6
J	1.46	1.74	2.09	2.45	2.95
v_o	3975	3816	3661	3530	3389
E_{1000}	107	148	204	269	368
Case Volume, V_c (cu. in.)	.152	.179	.211	.244	.287

Since we are only concerned with the Option I solution, the down-range data shown is only for 1,000 meters. Table D-1 shows that all rounds for all calibers satisfy the height of trajectory requirement of 12 inches or less over 300 meters. Hence, the curve for the H bound is not shown.

The MER-E bound at 1,000 meters is developed in the same manner as 50,000 psi peak chamber pressure space. Since recoil impulse is a function of bullet weight and muzzle velocity only, the J bound goes through the same points (w, v_o), but its location relative to the peak chamber pressure space is changed because this space has shifted upwards.

The resulting common round solution space of 52,000 psi peak chamber pressure is shown in figure 8.

Of the five basic calibers, only the .243 caliber provides common round solutions.[1]

THE 48,000 psi PEAK CHAMBER PRESSURE-OPTION I SOLUTION SPACE

Finally, we examine the common round solution space if the peak chamber pressure (requirement (5) of table 2) is decreased to 48,000 psi.[2] The result is to shift the peak chamber pressure space downward, and the common round solution space down and to the right.

The velocity and energy data at 48,000 psi peak chamber pressure are shown in table 6.

Since we are concerned with the common round solution, the only down-range data shown is for 1,000 meters. Table 6 also contains height of trajectory values at 300 meters (H_{300}) from table D-3, which show that all rounds for all calibers satisfy the H_r requirement. Hence, this bound is not shown.

---

[1]The caliber (d) for any point (w, v_o) in the solution space can be calculated from formula A-55 (with ℓ = 22 inches) using $\tilde{p}$ = 23,830 from table A-9. The associated case volumes can be estimated by plotting the case volume values from table 5 on figure A-8.

[2]This is a substantial decrease in pressure involving the use of cases whose volumes are slightly smaller than M/S as can be seen by plotting the case volume values from table 6 on figure A-8. The result is to decrease the muzzle velocities.

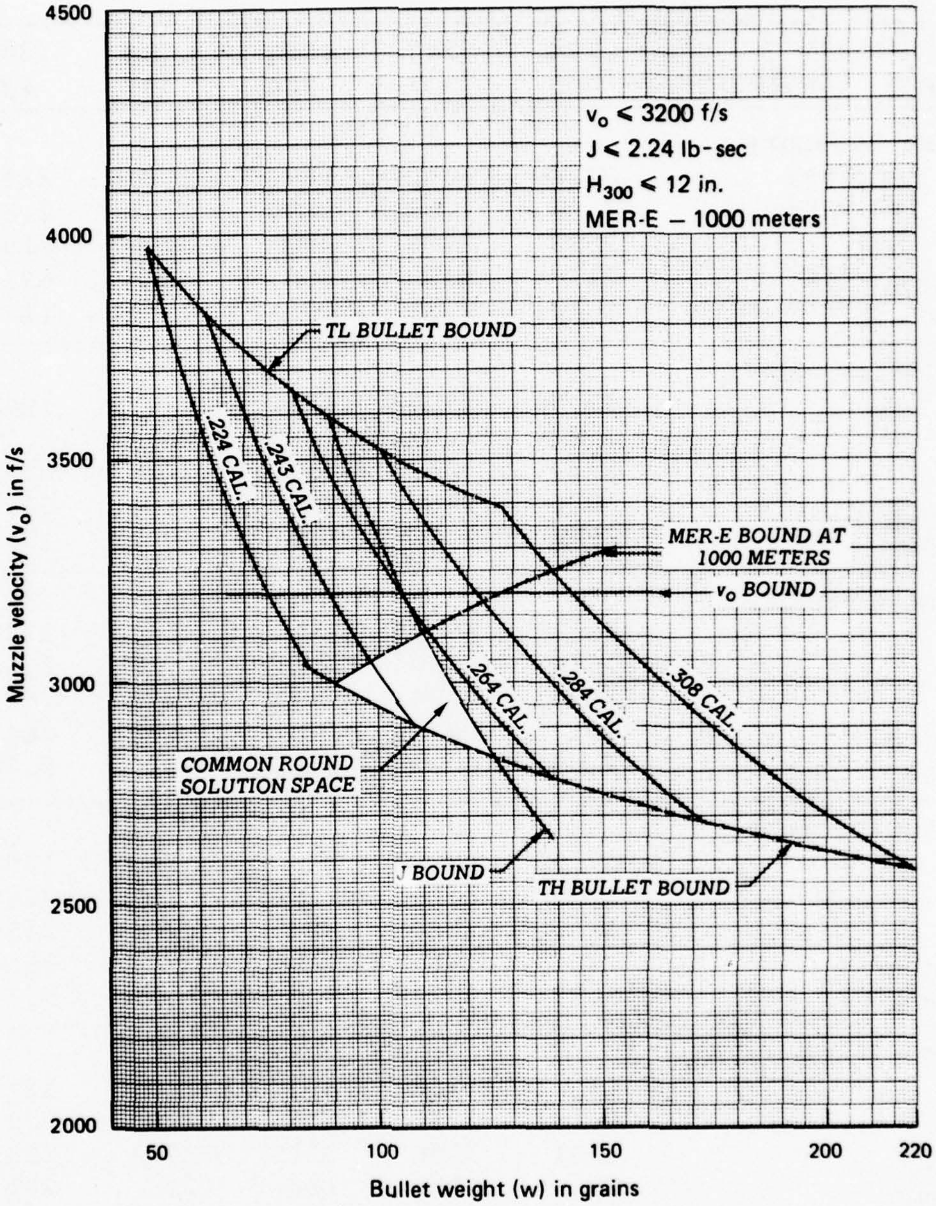

FIG. 8: COMMON ROUND SOLUTION SPACE, $\hat{p}$ = 52,000 psi

TABLE 6

VELOCITY AND ENERGY DATA FOR: PEAK CHAMBER PRESSURE-48,000 psi
BORE LENGTH-22 INCHES CASE CATEGORY-M/S

d (inches)	.224	.243	.264	.284	.308
MER-E (ft-lb)	305	333	364	396	435
TH Bullet Category					
w (grains)	84.40	107.7	138.2	172.0	219.4
J (lb-sec.)	1.35	1.63	1.98	2.36	2.86
v_o (fps)	2578	2475	2375	2290	2198
E_{1000} (ft-lb.)	217	287	380	485	631
H_{300} (inches)	9.14	9.73	10.39	11.04	11.80
M/H Bullet Category					
w	75.56	96.47	123.7	154.0	196.4
J	1.29	1.57	1.91	2.27	2.75
v_o	2725	2616	2510	2420	2323
E_{1000}	183	245	326	418	548
H_{300}	8.42	8.96	9.52	10.10	10.78
TM Bullet Category					
w	66.73	85.19	109.2	136.0	173.5
J	1.24	1.50	1.82	2.16	2.62
v_o	2899	2784	2671	2575	2472
E_{1000}	151	202	271	350	464
H_{300}	7.69	8.27	8.67	9.14	9.74
M/L Bullet Category					
w	57.90	73.92	94.78	118.0	150.5
J	1.18	1.43	1.74	2.06	2.49
v_o	3113	2989	2867	2764	2654
E_{1000}	120	162	218	283	376
H_{300}	6.97	7.36	7.81	8.25	8.76
TL Bullet Category					
w	49.07	62.64	80.33	100.0	127.6
J	1.13	1.36	1.64	1.95	2.35
v_o	3381	3246	3115	3003	2883
E_{1000}	89	122	164	216	291
H_{300}	6.26	6.59	6.96	7.32	7.76
Case Volume, V_c (cu. in.)	.088	.104	.123	.142	.167

The MER-E bound at 1,000 meters is developed in the same manner as for the 50,000 psi peak chamber pressure space. Since recoil impulse is a function of bullet weight and muzzle velocity only, the J bound goes through the same points (w, v_o), but its location relative to the peak chamber pressure space is changed because this space has shifted downwards.

The resulting common round solution space at 48,000 psi peak chamber pressure is shown in figure 9.

Of the five basic calibers only two, the .264 and .284 calibers, provide common round solutions and the .284 caliber contribution is very limited.[1]

The Option I Solution Space (48,000 to 52,000 psi Peak Chamber Pressure

A fairly accurate estimate of the common round solution space for peak chamber pressures over the range 48,000 to 52,000 psi can be obtained by drawing the three Option I (common round) solution spaces on a common set of (w, v_o) axes and connecting corresponding extreme points of these (triangular shaped) solution spaces. The resulting combined space is shown in figure 10.

The bullet weights interior to the resulting envelope range from approximately 90 to 161 grains. The calibers range from .229 to .284. However, while the .229 caliber employs the 90-grain bullet, the heaviest bullet for the .284 caliber is approximately 150 grains. The 161-grain bullet is the TH bullet for .278 caliber. Calibers (.229 and .278) are calculated using formula A-55 with ℓ = 22 inches and the proper value of $\tilde{p}$.

The solution space, shown in figure 10, represents the envelope generated by Option I solution spaces for the range of pressures 48,000 psi $\leq \hat{p} \leq$ 52,000 psi. The right bound (J = 2.24 lb-sec) can be plotted from formula C-4.[2] The upper bound of this envelope is the MER-E bound at 1,000 meters for 52,000 psi. The lower bound is the TH bullet bound associated with 48,000 psi. The left bound represents the trace of the intersection

[1] The caliber (d) for any point (w, v_o) in the solution space can be calculated from formula A-55 (with ℓ = 22 inches) using $\tilde{p}$ = 17,240 psi from table A-9. The associated case volumes can be estimated by plotting the case volume values of table 6 on figure A-8.

[2] Solve formula C-4 for w and set J = 2.24 lb-sec.

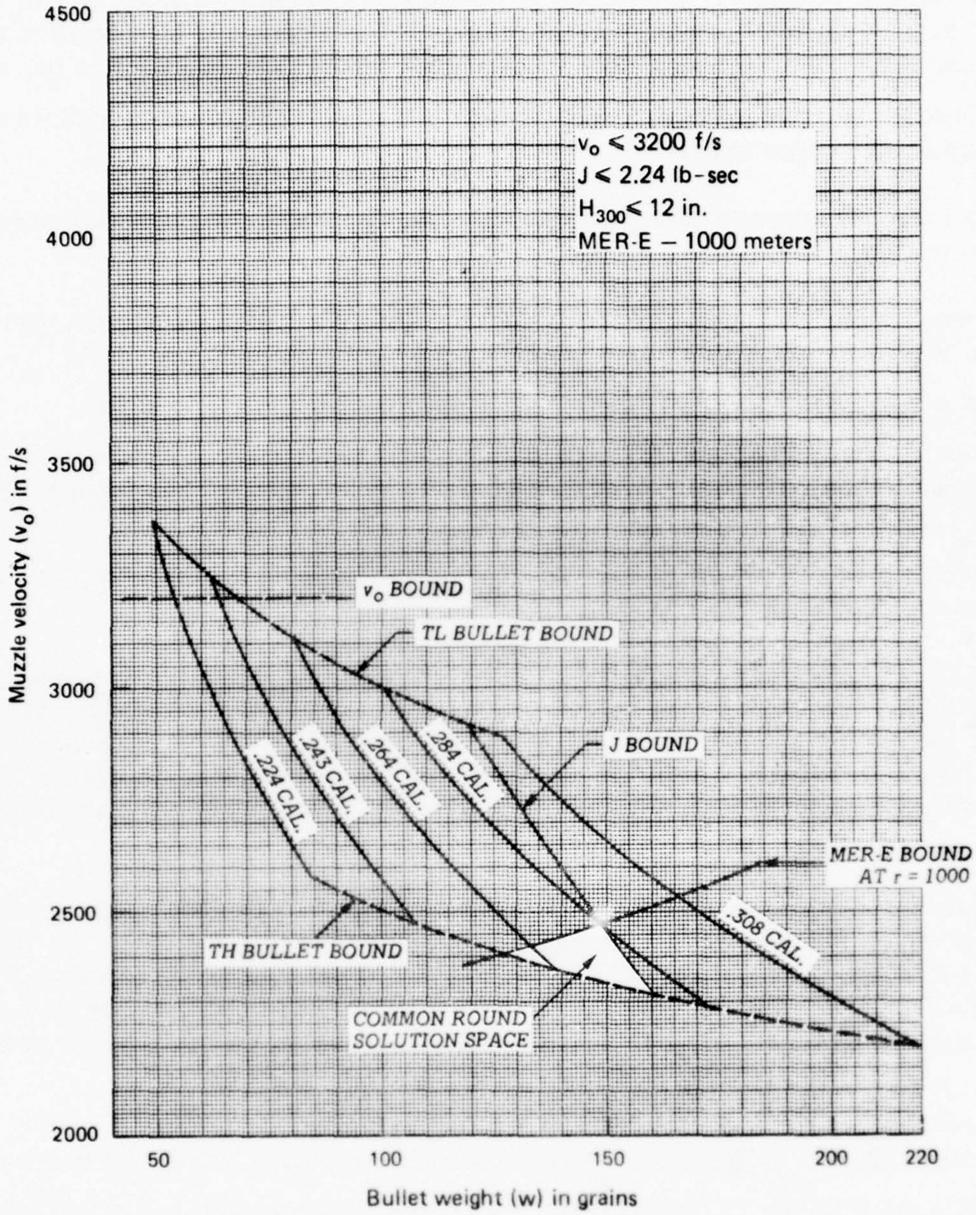

FIG. 9: COMMON ROUND SOLUTION SPACE, $\hat{p}$ = 48,000 psi

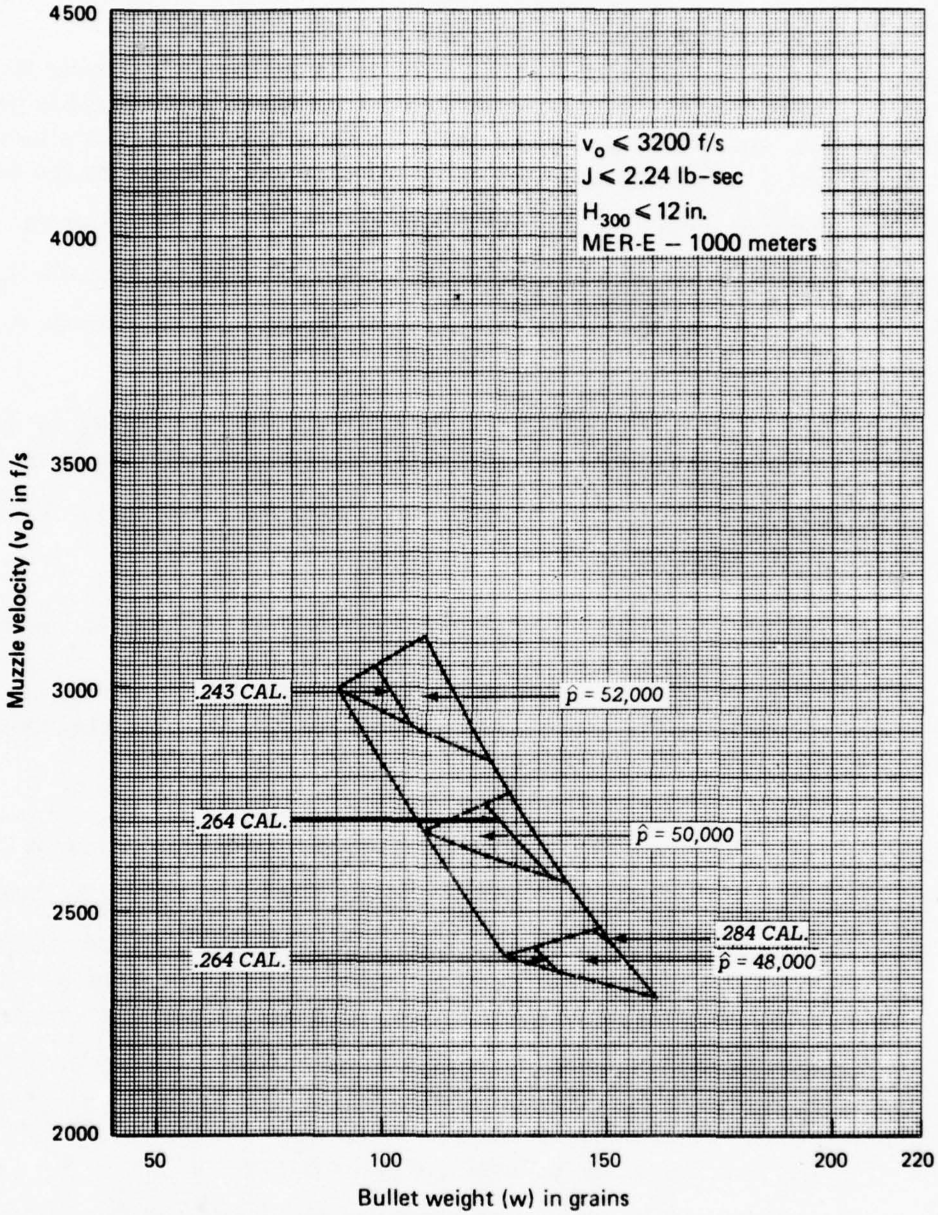

FIG. 10: COMMON ROUND SOLUTION SPACE,
48,000 psi $\leq \hat{p} \leq$ 52,000 psi

of MER-E bounds at 1000 meters with TH bullet bounds for peak chamber pressures over the range from 48,000 to 52,000 psi.[1]

Effect Of Reduced Height Of Trajectory And Recoil Impulse Requirements

The requirement that the height of the trajectory be 12 inches or less over 300 meters has not had any effect on the solution spaces for the three peak chamber pressures (48,000, 50,000, and 52,000 psi) examined. To show how to handle a situation where reducing H_r and J affect the common round solution space, consider the 48,000 psi peak chamber pressure level with the requirements: $H_{300} \leq 10.25$ inches and $J \leq 2.00$ lb-sec. Examination of the H_{300} values given in table 6 shows that the $H_{300} = 10.25$ inches bound intersects the $\hat{p} = 48,000$ psi space because some of these values exceed 10.25 inches.

Determination of the H-bound is carried out in a manner similar to that for determining a J bound. The H_{300} values from table 6 ($\hat{p} = 48,000$ psi) are plotted as a function of w for each of the five basic calibers and a horizontal line $H_{300} = 10.25$ inches is drawn as shown in figure 11.[2]

Next, the bullet weight and caliber values from figure 11 are located on the 48,000 psi space and connected to form a curve representing the H bound.

The J bound is constructed in the same manner as for figure 5, but the numerical values are different.

---

[1] Calibers (d) associated with points (w, v_o) of the envelope are no longer unique. For example, the point (w = 135 grains, v_o = 2600 fps) lies interior to the 50,000 psi space. However, this point also lies interior to other Option I solution spaces, such as 49,400 or 49,600 psi, but the associated calibers are not equal. Again, the point (w = 140 grains, v_o = 2500 fps) determines several calibers, each associated with a particular value of $\hat{p}$ lying between 48,000 and 50,000 psi. These calibers can be calculated using formula A-55, but they require determination of the $\tilde{p}$ associated with each proposed $\hat{p}$ value. In addition, the results must be checked to ensure that they do not violate MER-E or TH bounds for the $\hat{p}$ level and caliber involved.

[2] For a given caliber, height of the trajectory increases with bullet weight. In other words, for the 300-meter range, the high muzzle velocities of light bullets dominate the high ballistic coefficients of heavy bullets to result in shorter times of flight and flatter trajectories.

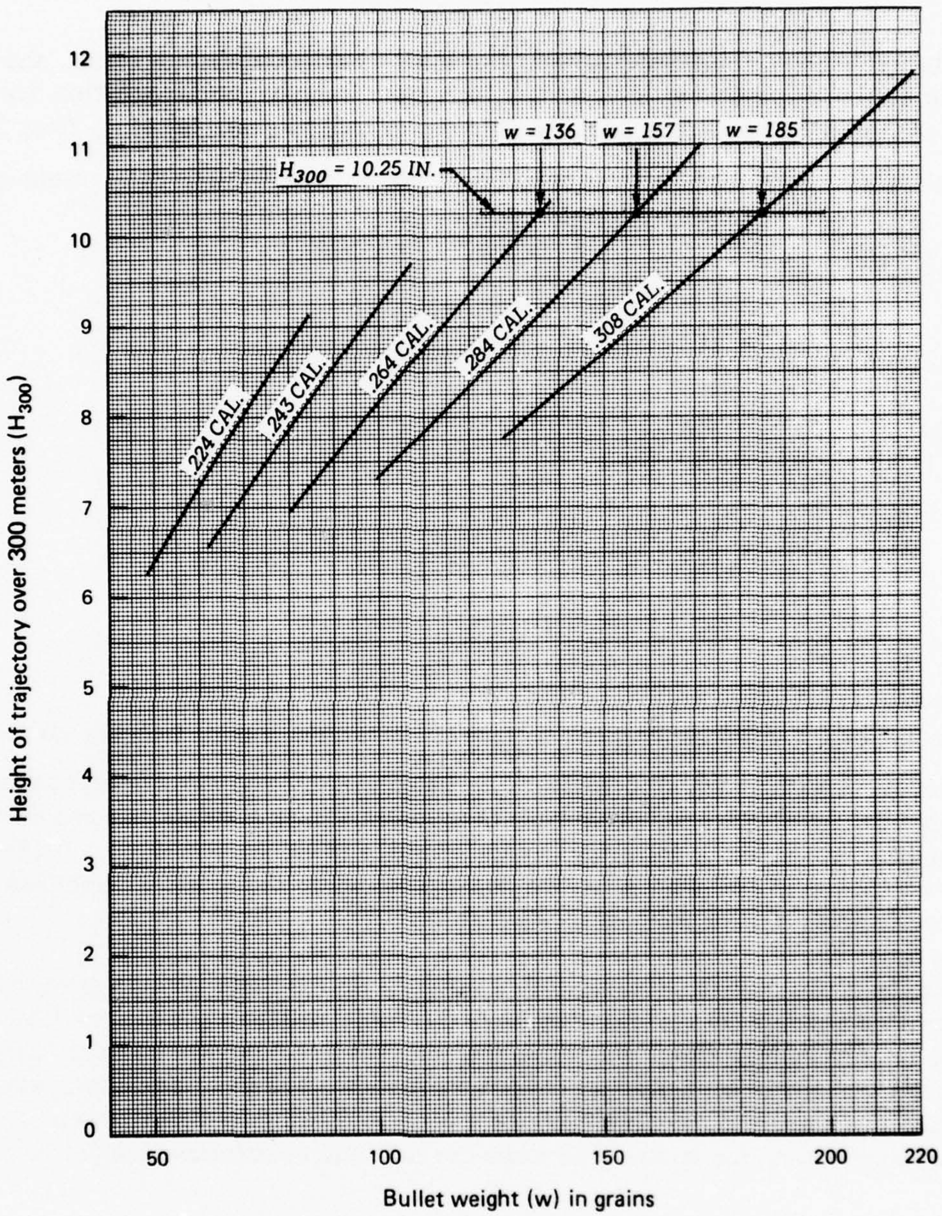

FIG. 11: H_{300} AS A FUNCTION OF BULLET WEIGHT (w) FOR GIVEN CALIBER (d), $\hat{p}$ = 48,000 psi

The resulting solution space is shown in figure 12. Both the new J = 2.00 pound-seconds bound and the old J = 2.24 pound-seconds curve are shown for comparison with figure 9.

The solution space is bounded above by the MER-E at 1000 meters bound, the new J bound, the TH bound, and the H bound.[1] Note that (relative to the solution space of figure 9) the solution space has been roughly cut in half by reducing H_{300} from 12 to 10.25 inches and more than half again by reducing J from 2.24 to 2.00 pound-seconds.

---

[1]The "smooth curve," representing the H_{300} = 10.25 inches bound, is a horizontal line. In other words, the three (w, d) combinations found in figure 11 all employ the same muzzle velocity. (Note that the three bullets involved all hit a target located at a 300-meter range and all have the same maximum ordinate of trajectory H_{300} = 10.25 inches.) Formulas developed in appendix A can be used to calculate the muzzle velocities and ballistic coefficients ($\overline{C}$) for each of these rounds.

The results of the calculations show that not only do the three rounds have the same muzzle velocity (2394 fps) but they have the same ballistic coefficient (.527 pounds per square inch). Thus, the three trajectories are a perfect match. It should be pointed out that while such perfect trajectory matches are possible among small-arms rounds, they are not possible between a large caliber (say 90mm) round and a small-arms round used for spotting because the ballistic coefficients cannot be made equal.

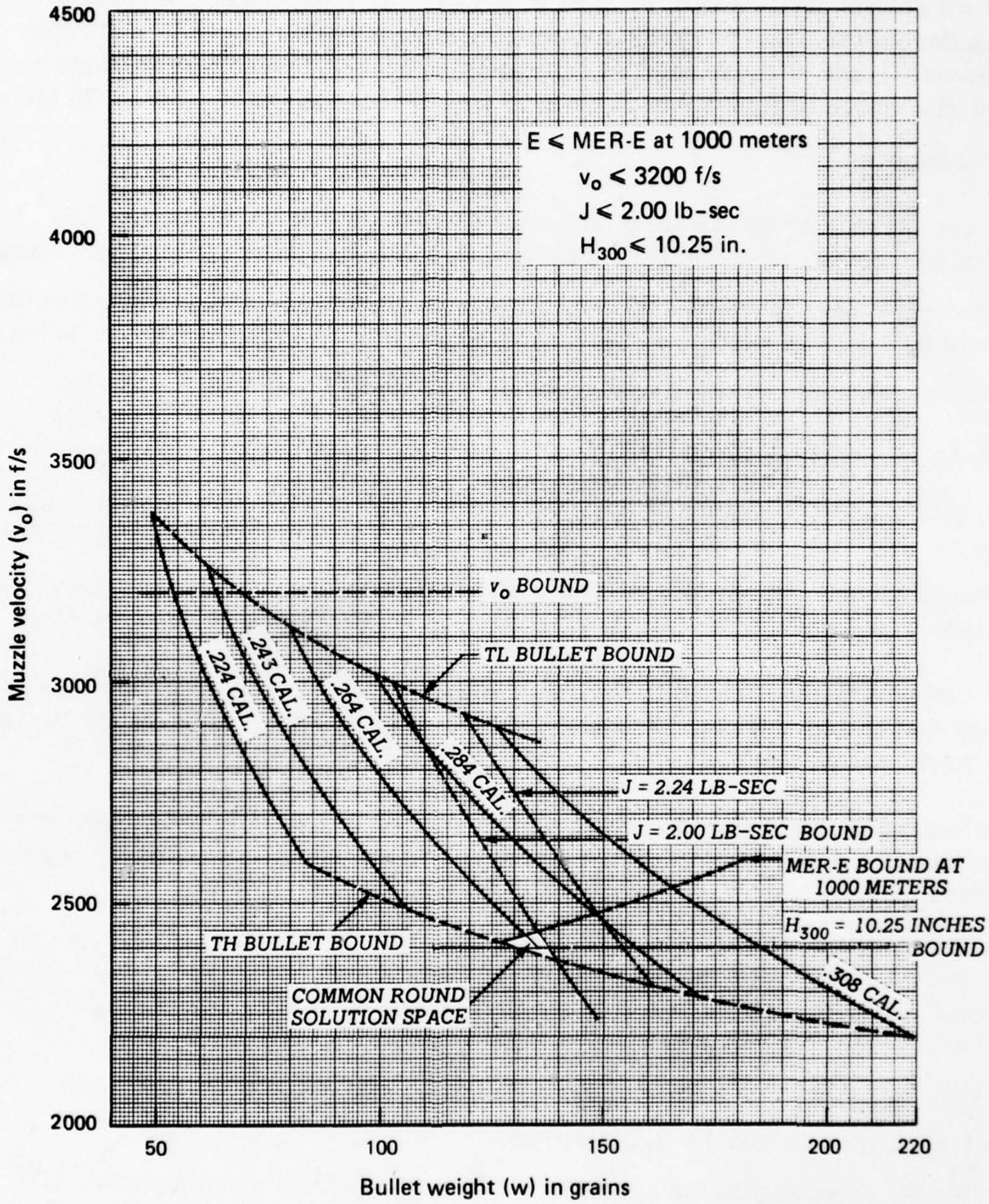

FIG. 12: EFFECT OF REDUCED RECOIL IMPULSE AND HEIGHT OF TRAJECTORY REQUIREMENTS COMMON ROUND SOLUTION SPACE, $\hat{p}$ = 48,000 psi

TRADE OFFS

The solution space, generated by a set of requirements acting as constraints, consists of all rounds not ruled out by laws of physics or "established conventional small-arms design practice." There are certain trade offs that can be made within solution spaces. Some of these can be illustrated by figure 13, which represents the Option I solution space of figure 9 with J = 2.00 pound-seconds and H_{300} = 10.25 inches curves superimposed.

There are no rifle weight (W) and free recoil (R) trade offs along the J = 2.24[1] bound, but H_r and MER values can be traded off along this constant J bound. However, along the J = 2.00 pound-seconds curve, W and R values can be traded off without exceeding their requirements bounds,[2] while H_r and MER values can each be traded for (w, v_o) combinations. Similar comments hold for trade offs along caliber H and MER-E curves.

In general, every point (w, v_o) in the solution space lies on the intersection of a set of caliber, MER-E, and H curves so that their values can be traded off by selecting different (w, v_o) points. However, practical considerations of trade-off possibilities are probably easier to understand than such theoretical ones as those above. These practical considerations will be left to the military personnel who generate the requirements.

There are also trade offs within the set of requirements. Such trade offs generate new solution spaces and will not, in general, be resorted to unless solution space trade offs do not prove satisfactory.

The selection of one of the Options (I, II, III) constitutes a trade off within the requirements because this selection makes the requirements more precise and also involves logistics trade offs.

The common round solution space at a peak chamber pressure of 50,000 psi shown in figure 5 is rather small. It contains only one of the five basic calibers, i.e., .264 caliber. From formula A-55, the left vertex of this solution space is associated with .244 caliber, the upper vertex with.273 caliber, and the right vertex with .266 caliber.

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[1] For checking purposes, use J = 2.244 lb-sec.

[2] Permissible W, R trade offs for any fixed J < 2.24 lb-sec. lie along a rectangular hyperbola in the W, R plane.

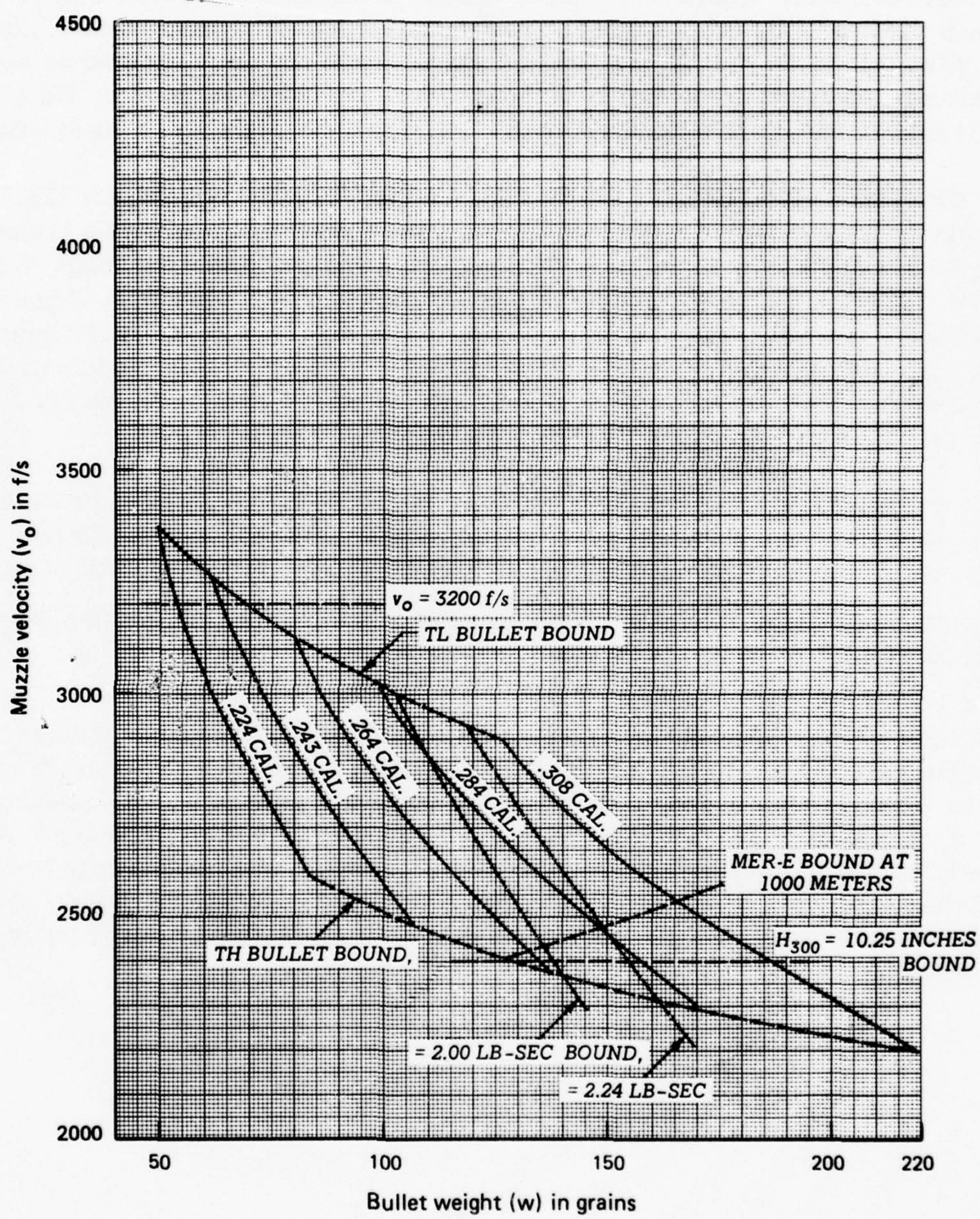

FIG. 13: TRADE OFFS WITHIN OPTION I SOLUTION SPACE, $\hat{p}$ = 48,000 psi

Thus, all calibers from .244 to .273 are included in the solution space of figure 5. There may be some cost advantage in selecting a caliber in common use such as one of the five basic calibers. There is one other caliber in common use which falls in the range from .244 to .273, besides .264. It is the so-called 25 caliber with d = .257 inches. Also, since the bounds of a solution space should not be considered as precisely defined because they are based on theory involving idealized powder, the 6-mm. (d = .243) caliber and the 270 Winchester (d = .277) caliber might merit consideration.

The admissible bullet weights for the .264 caliber (6.5mm) range from 122.5 to 138.2 grains. If the decision maker selects the .264 caliber because it is a rather common caliber and lies in the middle portion of the solution space, the range of 122.5 grains $\leq w \leq$ 138.2 grain values can be traded off against recoil impulse 2.12 pound-seconds $\leq J \leq$ 2.21 pound-seconds, and the final decision might well be to select a 130-grain bullet in .264 caliber with a muzzle velocity of 2,662 fps and a recoil impulse of 2.16 pound-seconds.[1] This round would yield 8.34 foot-pounds of free recoil energy in a 9-lb. rifle or 8.83 foot-pounds of free recoil energy in a 8.5-lb. rifle.

Rifle weight (W) and free recoil (R) can still be traded off at the fixed J = 2.16 pound-seconds level as discussed above. Such trade offs will generate trade offs involving maximum effective range (MER) and height of trajectory (H_r). All of the system parameter values are interrelated, but the primary trade offs involve W versus R , MER versus d, and H_r versus v_o.

It is rather evident that the sum total of trade offs open to the decision maker (even with a rather detailed set of requirements and resulting small solution space) is quite extensive. Therefore, while the metholology developed quantifies the solution process and shows the effects of specific requirements on the solution space, it does not relieve the decision maker of his primary responsibility. However, the methodology presented here quantifies the whole process and provides a means of reducing the amount of engineering and field testing to an acceptable level, while at the same time providing caliber comparisons.

[1]From formulas A-58 and A-43, the estimated weight of 100 such rounds is 4.5 pounds.

OBSERVATIONS

Some careful research involving theory and experimenting should be devoted to the problem of dealing with the maximum effective range (MER) requirement.

Measures of effectiveness developed in connection with maximum effective range determination should involve the bullet's capability to surrender energy in media more dense than air. These research efforts should not use bullets that are hollow point or soft point or bullets that tumble quickly in the more dense media. Neither should these efforts become involved in lethality (when bullets strike bone structure of vital areas), but should focus on caliber, bullet weight, and velocity for acceptable round designs.

The initial consideration in dealing with lethality, in the sense of energy transfer (from bullet to target) should be to determine if a bullet striking a media similar to body tissue surrenders energy in a fashion similar to its surrender in air. In other words, does the medium retardation increase substantially (and if so, by what law) when the velocity is increased.

Much competent opinion and considerable evidence indicate that the .224-caliber (5.56mm) round is deficient in energy at "longer ranges," while the .308-caliber (7.62mm) round has too high a recoil impulse to serve as a common round for the individual rifle and the machine gun.[1]

In selecting a round (caliber, bullet weight, muzzle velocity) based on the analysis of this paper, extremes should be avoided. In other words, stay away from the boundaries of the solution spaces. Experience indicates that the least desirable extremes are the opposite combinations of "heavy bullets in small cases" and "light bullets in large cases." In general, commercial as well as military rounds have not employed such opposite extremes.

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[1]Many experts feel that the .284 caliber (7mm) is ideal in the sense that it provides bullets having good sectional densities and high ballistic coefficients in the weight ranges, which experience indicates are desirable.

REFERENCES

1. Mellonics Systems Development Division of Litton Systems, Inc., "Comparative Analysis of Area Fire Versus Point Fire - Flat Trajectory Small-Arms Weapon System (U)," Secret, 31 May 1973

APPENDIX A

WEAPON ROUND PARAMETER RELATIONS

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WEAPON ROUND PARAMETER RELATIONS

INTRODUCTION

This appendix develops relations among parameters associated with small-arms weapons and rounds. In general, the parameters of concern are those that are required for comparisons involving calibers, bullet weights, and muzzle velocities. Thus, most of the material deals with topics that come under the heading of "interior ballistics."

By the word caliber, we mean the bullet diameter (d) measured in inches. Five calibers are employed to provide the basis for the numerical part of the analysis. These are: d = .224, .243, .264, .284, and .308 inches. Each of these calibers has a millimeter designation frequently found in small-arms literature. These designations are: 5.56, 6.0, 6.5, 7.0, and 7.62mm, respectively. If 25.4mm to the inch is used to convert one set of numbers to the other, it becomes evident that the above millimeter designations do not represent bullet diameters.

Historically, many schemes have been used to designate rounds of various calibers. Some denote the size of the hole bored in forming the barrel (before rifling). Others include the year of adoption by the military or the muzzle velocity of a certain bullet. Some involve case length, powder charge, or bullet weight, while still others designate the parent case from which a new round of different caliber had been formed. The proliferation of round names, where the cases are different in size and shape but the bullet diameters are the same, is most extensive in the so-called "22- and 30-caliber centerfire rounds." There are more than a dozen rather well-known "22-caliber" centerfire rounds having cases of different sizes and shapes but all having a bullet diameter d = .224 inches. The same situation exists for "30-caliber" rounds having a bullet diameter d = .308 inches.

The situation is further complicated by the fact that some rounds having different names are identical or use the same case. The M16, 5.56-mm. round is a "22-caliber" round that employs the same case as the 223 Remington round. Both use bullets that are .224 inches in diameter. The M14, 7.62-mm. round is a "30-caliber" round that employs the same case as the 308 Winchester round. Both use bullets that are .308 inches in diameter.

The only difference between the 244 and 6-mm. Remington rounds is that the 6mm is commercially loaded with a 100-grain bullet; the cases are the same, and the bullets are .243 inches in diameter.

Because of all this confusion over names, the only simple way to refer to caliber is by bullet diameter (d).

Both the classified and open literature dealing with small arms contain numerous comparisons of specific rounds fired in specific weapons but followed by inferences drawn in a fashion to indicate that the two calibers are being compared. Data on the M16 and M14 rifles does not provide a basis for comparison of 22 (d = .224) versus 30 (d = .308) calibers, but a comparison of a particular 22-caliber round with a particular 30-caliber round.

This appendix develops mathematical relations and numerical values associated with weapon and round parameters. The primary purpose of this development is to provide a basis for small-arms round comparisons.

In developing these mathematical relations, the condition of direct proportionality of y to x is expressed in three different ways depending on how it is to be used. These three expressions are:

$$y \propto x,$$

$$y = K x \quad , \text{K a constant,}$$

and

$$\frac{y_1}{y_2} = \frac{x_1}{x_2}$$

Corresponding expressions are used to denote indirect proportionality. The notation,

$$y \propto 1 \,,$$

is frequently used to denote that y is constant.

PARAMETER DEFINITIONS

Bullet Parameters

The primary bullet parameters are:

d = caliber taken equal to diameter in inches.

w = weight in grains.

i = form factor.

The form factor (i) is not determined by bullet geometry alone but is associated with a particular ballistic table. This is because the form factor serves as a multiplier, to relate the bullet under consideration to the standard projectile used in developing the particular ballistic table.

The sectional density (s) of a bullet is defined as the ratio of its weight in pounds to the square of its diameter in inches. Since there are 7000 grains in a pound,

$$s = \frac{w}{7000\ d^2} \quad . \tag{A-1}$$

The ballistic coefficient (C), rather than the form factor, is the parameter commonly used to measure a bullet's ability to maintain its velocity in air. The ballistic coefficient is defined by:

$$C = \frac{s}{i} \ ; \tag{A-2}$$

and, since i is relative to a particular ballistic table, C is also relative to that same ballistic table. Substituting for s, the ballistic coefficient can be written as:

$$C = \frac{w}{7000\ i\ d^2} \quad . \tag{A-3}$$

Since the form factor (i) is a ratio of retardations of a particular bullet to the standard projectile used to construct a ballistic table, it is without dimension. Therefore, the units for the ballistic coefficient (normally not stated) are pounds per square inch. The larger the ballistic coefficient, the lower the air drag on the bullet, and the better the bullet retains its velocity in air.

Case Parameters

The two primary case parameters are case volume or capacity (V_c) and caliber (d). The case volume is measured to the junction of the neck and shoulder in bottleneck cases. Case volume is usually measured by weighing the amount of water the case holds (when filled to the neck-shoulder junction) and converting the weight of water to volume. We define case volume in cubic inches. The case neck diameter is specified in terms of the caliber bullet for which it is formed.

Bore Parameters

The term bore refers to the interior of the barrel. The primary bore parameters are caliber (d) equal to bullet diameter, and length (ℓ) measured from the junction of the neck and shoulder to the muzzle of the barrel.

The volume of the bore (V_b) is defined as:

$$V_b = \frac{\pi}{4} d^2 \ell \;, \tag{A-4}$$

where d and ℓ are measured in inches, and V_b is in cubic inches.

Expansion Ratio

The expansion ratio (ρ) involves both the case and bore. It is the volume available to the propellant at the instant the bullet leaves the muzzle divided by the volume (case capacity) available at the instant of firing. This parameter is involved in the relation of peak chamber pressure to muzzle velocity.

The expansion ratio of a case-bore combination is defined by:

$$\rho = \frac{V_c + V_b}{V_c} \;, \tag{A-5}$$

where V_c and V_b are the volumes of the case and bore, respectively.

Muzzle Energy

The muzzle energy of a bullet (from the kinetic energy formula) is:

$$E_o = \tfrac{1}{2} m \,(v_o)^2 \;, \tag{A-6}$$

where m is the mass of the bullet, and v_o is its muzzle velocity.

If v_o is in feet per second, m is "pounds of mass" and E_o is in foot-pounds. Since there are 7000 grains in a pound,

$$E_o = \frac{1}{14,000g} w \,(v_o)^2 \;, \tag{A-7}$$

where w is bullet weight in grains, and g denotes the acceleration of gravity in feet per second per second.

Ballistic Efficiency

The ballistic efficiency (ε) of a round (when fired in a bore of length ℓ) is the ratio of the muzzle energy divided by the energy contained in the powder load of weight c. Since the energy in the powder is proportional to the powder weight, ε is proportional to E_o divided by c:

$$\varepsilon \propto \frac{E_o}{c} \quad . \tag{A-8}$$

If ε is constant, $v_o^2 \propto c/w$ and conversely.

FRY'S BASIC RELATION

Macon Fry, formerly on the staff of the Operations Research Office, derived a basic relation which is applicable to both between-caliber and within-caliber comparisons. This relation (contained in an unpublished paper) is part of an analysis to determine the muzzle velocity which would maximize the range at which a preselected velocity occurs under constraints of fixed bore length and fixed mean effective pressure. Excerpts from Fry's analysis, with slight notation changes and minor modifications, follow.

From formula A-7, E_o is proportional to the bullet weight multiplied by the square of the muzzle velocity,

$$E_o \propto w\,(v_o)^2 \quad . \tag{A-9}$$

Since muzzle energy is the result of force acting on the bullet mass over the length of the bore, it can be expressed as the mean effective force ($\tilde{F}$) times bore length (ℓ). Mean effective force equals the mean effective pressure ($\tilde{p}$) times the bullet cross section area. Hence,

$$E_o = \tilde{F}\,\ell \propto \tilde{p}\,d^2\,\ell \quad . \tag{A-10}$$

Eliminating E_o between A-9 and A-10 yields:

$$w\,v_o^2 \propto \tilde{p}\,d^2\,\ell \quad . \tag{A-11}$$

For round comparisons, $\tilde{p}$ and ℓ should be held constant. Hence, A-11 reduces to:

$$w\, v_o^2 \propto d^2 \quad . \tag{A-12}$$

We shall refer to A-12 as Fry's Basic Relation.

COMPARISONS INVOLVING TWO CALIBERS

In comparing rounds of different caliber (between-caliber comparisons), it is frequently stated that the bullets must be homologous. This implies that there is a "likeness of structure" in the bullets of various calibers so that bias is not introduced into the comparison. For purposes of the present analysis, we shall require that the bullets, cases, and rounds satisfy homology conditions. Furthermore, the weapons in which the rounds are fired must have barrels such that the case-bore combinations are homologous. Finally, the mean effective pressures must be constant.

The term homologous will be applied only to weapon-round components where two (or more) calibers are involved. The remainder of this section is devoted to translating the above homology requirements into relations among small-arms parameters of different calibers.

Homologous Bullets

In seeking differences in bullet performance which result from difference in their calibers, it is important to consider only bullet designs where these differences in performance are solely the result of caliber difference. Thus, corresponding bullet dimensions should be in the same ratio as their diameters. In other words, corresponding dimensions should be scaled by caliber (d). The form factor (i) should be constant and the bullet weights (w) proportional to the cubes of the diameters (d). Two bullets of different calibers will be called homologous if they possess these properties.

Hence, two bullets of calibers d_1 and d_2, having respective weights w_1 and w_2 and form factors i_1 and i_2, are homologous if:

$$i_1 = i_2 , \tag{A-13}$$

and

$$\frac{w_1}{w_2} = \left(\frac{d_1}{d_2}\right)^3 \quad . \tag{A-14}$$

Homologous Case-Bore Relations

The primary case-bore parameters are: caliber (d), bore length (ℓ), and case volume (V_c).

The volume of the bore (V_b) is given by:

$$V_b = \frac{\pi}{4} d^2 \ell \quad . \tag{A-15}$$

The expansion ratio (ρ) is defined as:

$$\rho = 1 + \frac{V_b}{V_c} \quad . \tag{A-16}$$

Since it is well known that changing the bore lengths, while holding other parameters fixed, results in a change in muzzle velocity (v_o), it is evident that round comparisons (of either the same or different calibers) should be made with ℓ held constant:

$$\ell \propto 1 \quad . \tag{A-17}$$

Therefore, from relations A-15 and A-17, the volume of the bore is proportional to the caliber squared:

$$V_b = K_1 d^2 \quad , \tag{A-18}$$

where

$$K_1 = \frac{\pi \ell}{4} \quad .$$

Since, in general, cases for large caliber rounds have greater capacity (V_c) than cases for small calibers, it is reasonable to require that case volume be proportional to some positive power (m) of caliber:

$$V_c = K_2 d^m, \; m > 0 \quad . \tag{A-19}$$

Hence, from relations A-16, A-18, and A-19,

$$(\rho - 1) = \frac{K_1}{K_2} d^{2-m} \quad . \tag{A-20}$$

However, this relation does not provide much insight into the value of m. Furthermore, just as V_b is proportional to d^2 and V_c is to be proportional to d^m, it is desirable to have ρ be proportional to some power (n) of d. On the other hand, if one considers

the definition of ρ, given by A-16 as a ratio of volumes available to propellant (in both solid and gaseous forms), it seems reasonable that ρ, just as ℓ, should be constant.

Since we are dealing with an idealized situation and estimates of case volume and peak chamber pressure will not hold exactly for various real powders, we assume that to a first approximation the muzzle energy of the bullet (E_o) is proportional to the amount of powder (c):

$$E_o \propto c \quad . \tag{A-21}$$

Also, since the expansion ratio affects the relation of peak chamber pressure to muzzle velocity (see figure A-3), we shall require that the same fraction of case volume be filled with powder for all rounds being compared. In other words, the amount of powder (c) is to be proportional to the case volume (V_c):

$$c \propto V_c \quad . \tag{A-22}$$

From (A-7), (A-12), (A-16), (A-17), (A-21), and (A-22), it follows that case volume is proportional to the square of caliber and the expansion ratio is constant:

$$V_c \propto d^2$$

and (A-23)

$$\rho \propto 1 \quad .$$

Hence, for comparisons of two calibers d_1 and d_2, the homologous bore conditions are:

$$\ell_1 = \ell_2$$

and

$$\frac{V_{b_1}}{V_{b_2}} = \left(\frac{d_1}{d_2}\right)^2 \quad ; \tag{A-24}$$

the homologous case condition is :

$$\frac{V_{c_1}}{V_{c_2}} = \left(\frac{d_1}{d_2}\right)^2 \quad ; \tag{A-25}$$

and the homologous case-bore relation is:

$$\rho_1 = \rho_2 \quad . \tag{A-26}$$

Homologous Rounds

Two rounds of respective calibers d_1 and d_2 are homologous if they employ homologous bullets in homologous cases and, when fired in rifles satisfying homologous case-bore conditions, develop equal mean effective pressures.

Summarizing the above homologous weapon-round parameter relations for comparing calibers d_1 and d_2, we have:

$$i_1 = i_2 \quad ,$$

$$\frac{w_1}{w_2} = \left(\frac{d_1}{d_2}\right)^3 \quad ,$$

$$\tilde{p}_1 = \tilde{p}_2 \quad ,$$

$$\ell_1 = \ell_2 \quad , \tag{A-27}$$

$$\rho_1 = \rho_2 \quad ,$$

$$\frac{V_{b_1}}{V_{b_2}} = \left(\frac{d_1}{d_2}\right)^3 \quad ,$$

and

$$\frac{V_{c_1}}{V_{c_2}} = \left(\frac{d_1}{d_2}\right)^3 \quad .$$

Derived Relations

The relations involving weapon-round parameters which are developed above are by no means independent. Several additional relations among parameters, which are useful in determining numerical values satisfying homology conditions, can be derived.

Formulas A-3, A-13, and A-14 imply that the ballistic coefficient is proportional to caliber:

$$\frac{C_1}{C_2} = \frac{d_1}{d_2} \quad . \tag{A-28}$$

Relations A-24 and A-25 imply that bore volume is directly proportional to case volume:

$$\frac{V_{b_1}}{V_{b_2}} = \frac{V_{c_1}}{V_{c_2}} \quad . \tag{A-29}$$

Relations A-12 and A-14 imply that the muzzle velocity is inversely proportional to the square root of caliber:

$$\frac{v_{o_1}}{v_{o_2}} = \sqrt{\frac{d_2}{d_1}} \quad . \tag{A-30}$$

Formulas A-28 and A-30 imply that muzzle velocity is inversely proportional to the square root of the ballistic coefficient:

$$\frac{v_{o_1}}{v_{o_2}} = \sqrt{\frac{C_2}{C_1}} \quad . \tag{A-31}$$

Conditions A-7 and A-12 imply that muzzle energy is directly proportional to caliber squared:

$$\frac{E_{o_1}}{E_{o_2}} = \left(\frac{d_1}{d_2}\right)^2 \quad . \tag{A-32}$$

Conditions A-32, A-24, and A-25 imply

$$\frac{E_{o_1}}{E_{o_2}} = \frac{V_{c_1}}{V_{c_2}} = \frac{V_{b_1}}{V_{b_2}} \quad . \tag{A-33}$$

From A-32 and A-8,

$$\frac{\epsilon_1}{\epsilon_2} = \frac{c_2}{c_1} \cdot \left(\frac{d_1}{d_2}\right)^2 \quad . \tag{A-34}$$

COMPARISONS WITH CALIBER FIXED

While between-caliber comparisons are of primary importance when dealing with the selection of a small-arms system, within-caliber comparisons (caliber fixed) immediately become involved if two or more bullet weights of the same caliber are being considered. Also, there are situations, such as the recent consideration of adopting a 68-grain bullet for the M16 round, where the only option involves a change in one component.

Within-caliber comparisons, where the case is fixed, are common considerations because they arise from simply changing bullet weights while retaining the original case and rifle. Thus, no case reforming or rifle chamber modifications are required. Therefore, such comparisons are, in fact, comparisons of bullets of the same caliber loaded in the same case and fired from the same rifle. These are the fixed caliber comparisons which we shall employ when between-caliber (homologous) comparisons become involved with more than one bullet weight for each caliber.

Denoting two bullet weights of the same caliber by w_1 and w_2 , many of the same relations hold for their associated parameters that held for between-caliber comparisons. Other relations simply involve replacement of d by w.

If we start out with the same relation forms as were used for between-caliber comparisons (except that the subscripts in the former case denoted two calibers, while here they denote two bullet weights of the same caliber), we have:

$$\begin{aligned} i_1 &= i_2 \ , \\ \tilde{p}_1 &= \tilde{p}_2 \ , \\ \ell_1 &= \ell_2 \ , \\ \rho_1 &= \rho_2 \ , \end{aligned} \qquad \text{(A-35)}$$

and

$$d_1 = d_2 \ .$$

Now, since d and ℓ are fixed, formula A-4 implies that V_b is constant:

$$V_{b_1} = V_{b_2} \ , \qquad \text{(A-36)}$$

But, if ρ and V_b are constant, the case volume is constant from A-16:

$$V_{c_1} = V_{c_2} \quad . \tag{A-37}$$

Now, the within-caliber comparisons that are employed as part of a set of between-caliber comparisons go one step beyond $V_{c_1} = V_{c_2}$ and require that the same case is used for all bullet weights of a given caliber. In other words, not only must case volumes be equal, but case geometries must be identical.

In the formula for ballistic coefficient A-3, i and d are both constant. Hence, for within-caliber comparisons, the ballistic coefficient is proportional to the bullet weight:

$$\frac{C_1}{C_2} = \frac{w_1}{w_2} \quad . \tag{A-38}$$

Fry's Basic Relation, $w\, v_o{}^2 \propto d^2$, holds for both between- and within-caliber comparisons. However, since d is fixed for within-caliber comparison, the product $w\left(v_o\right)^2$ is constant, and

$$\frac{v_{o_1}}{v_{o_2}} = \sqrt{\frac{w_2}{w_1}} \quad , \tag{A-39}$$

where w_1 and w_2 are the weights of two bullets of the same caliber, while v_{o_1} and v_{o_2} are the associated muzzle velocities (v_o) for these two bullets.

Since $w\left(v_o\right)^2$ is constant, the muzzle energies are constant:

$$E_{o_1} = E_{o_2} \quad . \tag{A-40}$$

SUMMARY OF BASIC PARAMETER RELATIONS

Following is a summary of the basic parameter relations, expressed in proportionality notation, where $x \propto 1$ denotes that x is constant.

Between-caliber	Within-caliber
--	$d \propto 1$
$i \propto 1$	$i \propto 1$
$\underset{\sim}{\ell} \propto 1$	$\underset{\sim}{\ell} \propto 1$
$p \propto 1$	$p \propto 1$
$\rho \propto 1$	$\rho \propto 1$
$w\, v_o{}^2 \propto d^2$	$w\, v_o{}^2 \propto 1$
$w \propto d^3$	--
$C \propto d$	$C \propto w$
$v_o{}^2 \propto 1/d$	$v_o{}^2 \propto 1/w$
$V_b \propto d^2$	$V_b \propto 1$
$V_c \propto d^2$	$V_c \propto 1$
$E_o \propto d^2$	$E_o \propto 1$
$\varepsilon \propto 1$	$\varepsilon \propto 1$

PRESSURE CONSIDERATIONS

The relation of chamber pressure to muzzle velocity is rather involved. When the propellant is ignited, the gases formed must build up sufficient pressure to force the bullet out of the case. This is sometimes referred to as "popping the cork." There may be some "free bore," which is the distance that the bullet travels before engaging the rifling. If the bullet contacts the lands (rifling) when the round is chambered, the free bore is zero; this condition is frequently described as long-loaded. When the round is fired, the lands of the rifling swage grooves in the bearing surface of the bullet, which is frequently referred to as engraving. After engraving, the bullet continues to move down the bore, its speed increasing with time (if the barrel is not too long) until it exits with muzzle velocity v_o. There are several factors that obviously affect the interior ballistics. Some of these are: the amount and brand of powder used, the caliber and weight of the bullet, and the length of the barrel. Other factors, perhaps not so obvious, also affect the internal ballistics. Among these are: the temperature of powder at ignition, the primer used, the geometry of the case, the fraction of the case volume occupied by powder, the expansion ratio, and the ratio of weight of the powder to the weight of the bullet.

While many of the problems associated with interior ballistics are most successfully dealt with through direct experiments, considerable mathematical theory does exist. Much of the mathematical theory is too complicated and detailed to be of use here. However, a simplified, somewhat heuristic description of the primary actions that are involved should contribute to an understanding of overall internal ballistics.

Discussions of interior ballistics usually introduce time-pressure or travel-pressure curves, which show the chamber pressure as a function of time after ignition or of bullet travel down the bore. Figure A-1 (showing the general shape of such curves) indicates the most critical time aspects of bullet travel from primer ignition to exit of the bullet from the muzzle.

Since the bullet is secured in the neck of the case by crimping, a sealing agent, or both, some pressure must be built up before the bullet is ejected from the case, which takes place at a time denoted by t_1. The bullet travels any free bore distance, engages the rifling, and is engraved. Only the bearing surface is engraved, and this process is completed at a time, denoted by t_2, during bullet travel equal to the length of the bearing surface.

While it would be preferable to have a more constant level of pressure than the "typical curve" shown in figure A-1, this is very difficult to achieve because the rate of powder burn is proportional to the amount of powder surface remaining, and the volume

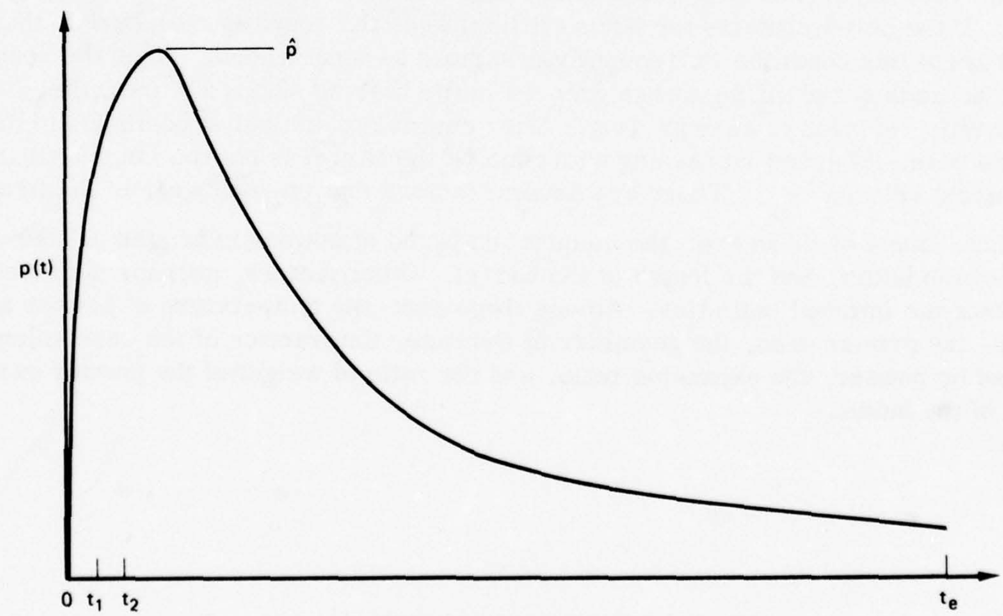

FIG. A-1: TYPICAL PRESSURE TIME CURVE

available to the gas is increasing in a non-linear fashion as the bullet moves down the bore. The result is a curve that peaks quickly (frequently before the bullet has travelled an inch) to pressure $\hat{p}$, and then decreases to a relatively low level considerably before the bullet exits from the barrel. The bullet exit time, denoted by t_e, is on the order of .001 second for moderate muzzle velocities such as 2600 feet per second.

It is desirable to have "propellant burn out" take place just prior to the bullet's leaving the muzzle. With some rounds, especially those of "small caliber" employing larger cases, the bullet leaves the muzzle before the powder is completely burned. Thus, some unburned powder is blown from the muzzle and the load is frequently referred to as over-bore.

It is the peak chamber pressure ($\hat{p}$) which determines how strong the weapon receiver must be in order to withstand the pressures generated. Building additional strength into a weapon tends to increase its cost and weight. Therefore, it is desirable that small-arms systems comparisons be carried out at constant peak chamber pressures. Aside from systems-cost consideration, it can be argued that down-range projectile energies, which are judged deficient, can be increased by "burning more powder" with an increase in the peak chamber pressure. While this is true, it is almost certain to violate the required condition that mean effective pressures be held constant. Small-arms literature contains numerous examples of round comparisons where the peak chamber pressures are not held constant. Such comparisons are valid in relation to the particular rounds being tested, or compared, but do not permit deductions relative to differences in parameter values which result from differences in caliber.

From an examination of pressure curves, such as figure A-1, it is evident that the peak chamber pressure ($\hat{p}$) is not affected by the bore length (ℓ) because peak chamber pressure occurs before the bullet has travelled more than approximately an inch or so down the bore. It is desirable that burn-out (complete combustion of the powder charge) occur just prior to the bullet's leaving the muzzle. If burn-out occurs too soon before bullet exit, the friction between bore surface and bullet bearing surface may exceed the force from the propellant gases, with the result that the bullet slows down. However, there is little danger of this occurring with modern centerfire systems.

The mean effective pressure ($\tilde{p}$) is not the mean of the chamber pressure curve shown in figure A-1. The chamber pressure must overcome resistance to bullet motion (such as bore friction) before the bullet can be accelerated. The effective pressure (p_e) is the working pressure making up the force which is involved in Newton's second law:

$$\left(p_e \frac{\pi d^2}{4}\right) = \frac{1}{7000} m a = \frac{w}{7000g} \frac{dv}{dt} \quad . \qquad \text{(A-41)}$$

It follows that the mean effective pressure (pounds/square inch) is:

$$\tilde{p} = \frac{48\ E_o}{\pi d^2 \ell} = \frac{w\ v_o^2}{9380\ \pi d^2 \ell} \quad . \tag{A-42}$$

Thus, $\tilde{p}$ can be calculated from other system parameters. Unfortunately, $\hat{p}$ cannot. Even if the caliber, bullet weight, muzzle velocity, case volume, and bore length are known, the value of $\hat{p}$ can still vary with the particular powder and primer and the forces required to pop the bullet from the case and to engrave its bearing surface.

However, these factors affecting $\hat{p}$ are not of direct concern for purposes of developing the first approximation relations of this paper. Furthermore, in later sections of this appendix, data will be developed which indicates that (in theory) $\hat{p}$ will be constant if $\tilde{p}$ is held constant.

NOMOGRAPHS

As stated above (under PRESSURE CONSIDERATIONS), it is highly desirable that round comparisons are carried out with equal peak chamber pressures. The between- and within-caliber parameter relations developed above (under COMPARISONS INVOLVING TWO CALIBERS and COMPARISONS WITH CALIBER FIXED), require that the mean effective pressures ($\tilde{p}$) be held constant. TECHNIK DRAWING No. TR 63-71[1] provides a means of checking on the peak chamber pressures. This drawing is shown in figure A-2 and will hereafter be referred to as the TECHNIK Nomograph. It involves four parameter values: the peak chamber pressure ($\hat{p}$), the muzzle velocity (v_o), the expansion ratio (ρ), and the weight ratio of powder to bullet (c/w). In theory, if any three of these values are known, the fourth can be estimated from the nomograph by drawing a horizontal line from a known point on one side to the appropriate curve on the other side.

For small-arms only a subregion of the TECHNIK Nomograph is required. This subregion is shown in figure A-3. In some of the applications which follow, it is necessary to consider cases which are "just filled with powder" (to the junction of the shoulder and neck of the case). Thus, V_c is just large enough to hold the powder of weight c. To determine this relation, it is necessary to assign an idealized powder loading density. After consulting with BRL and the technical staff of The American Rifleman, the idealized powder was assigned a loading density of 230 grains per cubic inch.[2]

[1] Copies available from Frankford Arsenal.

[2] This is slightly more than the bulk density (0.900 grams per cubic centimeter) of a common powder known as IMR 4895, which is a very versatile powder being employed for handloading rifle rounds ranging from .172 to .460 caliber.

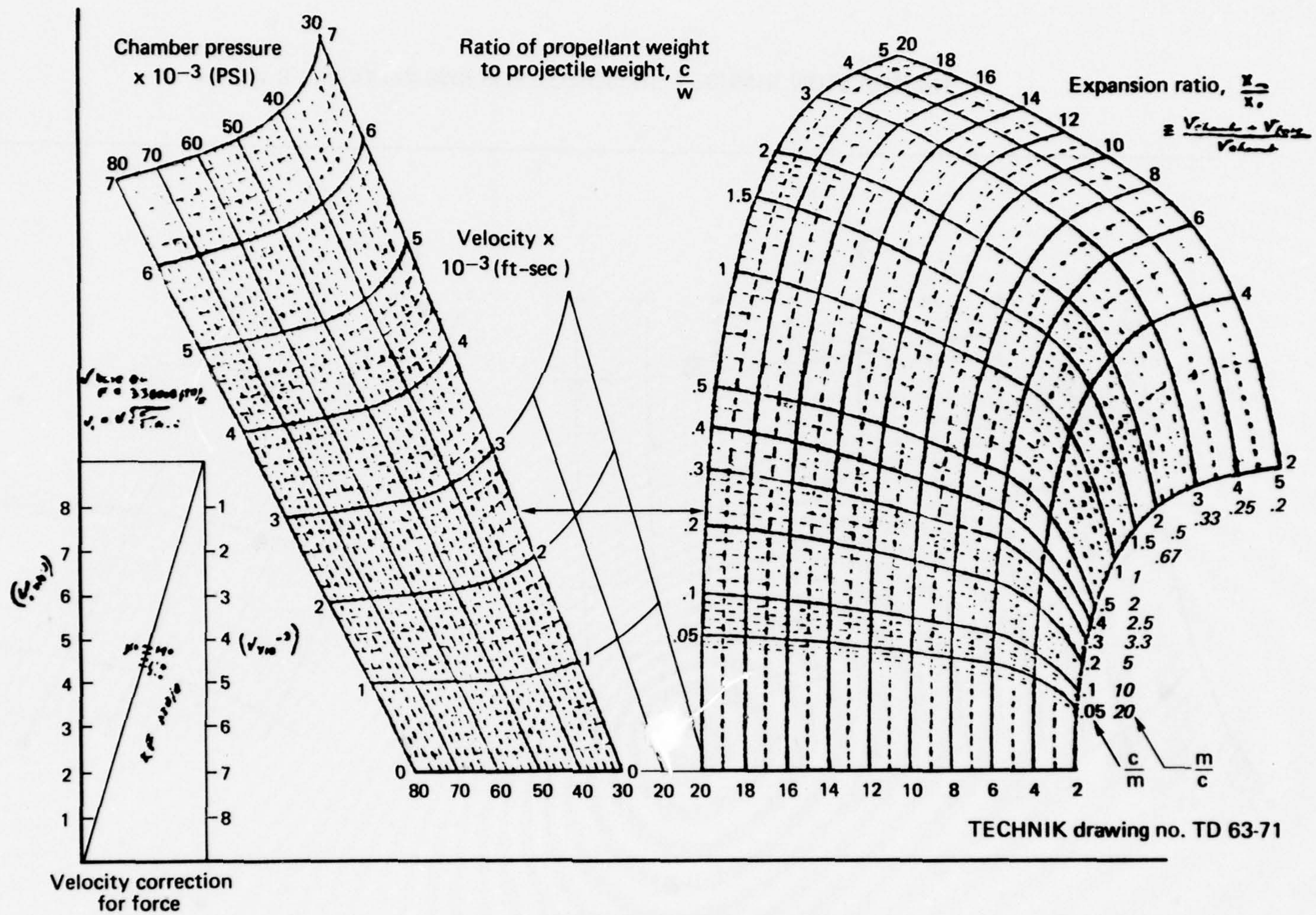

FIG. A-2: TECHNIK NOMOGRAPH

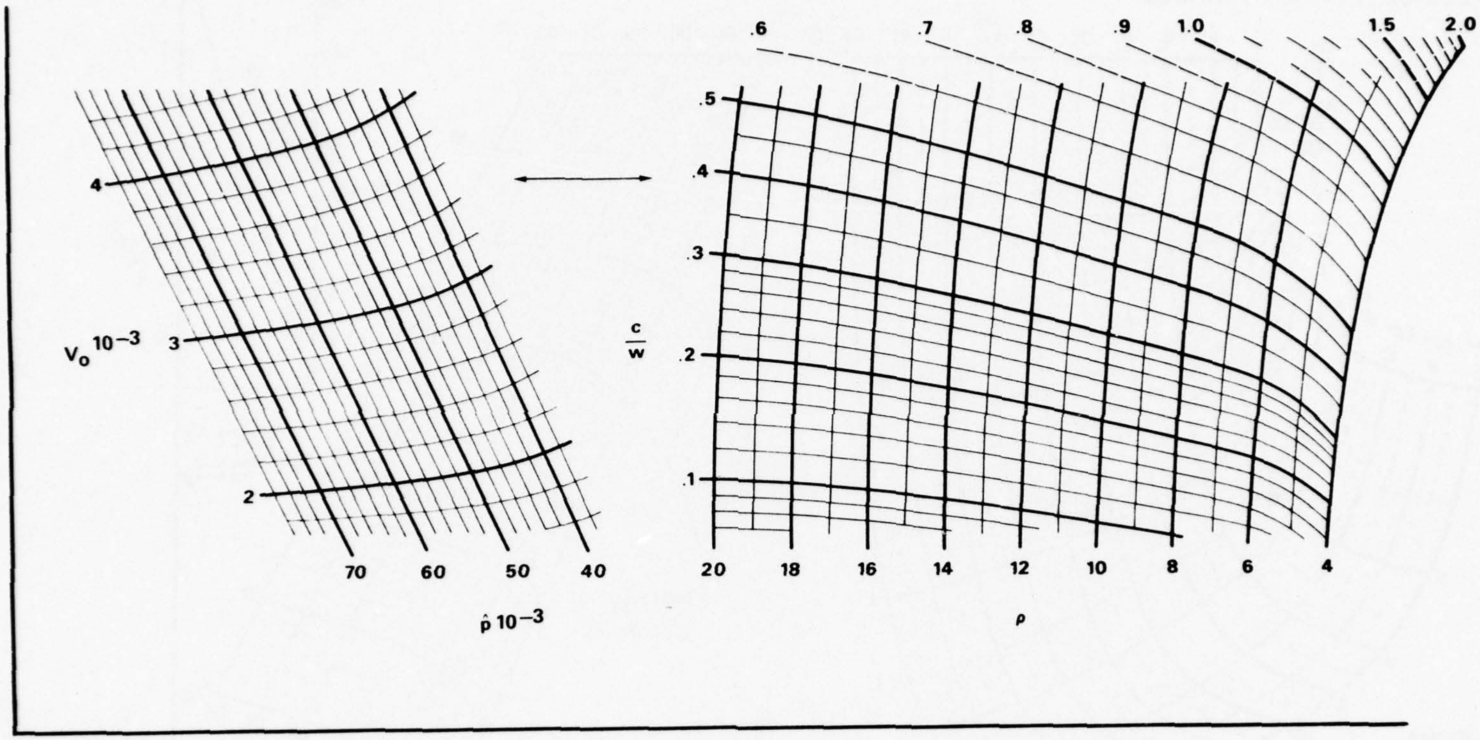

FIG. A-3: SMALL-ARMS SUBREGION OF TECHNIK NOMOGRAPH

Taking 230 grains per cubic inch as the loading density, we have

$$c = 230\ V_c \quad . \tag{A-43}$$

Substituting for V_c from expression A-43 into formula A-16 yields

$$\rho = 1 + \frac{230\ V_b}{c} \quad . \tag{A-44}$$

Hence,

$$c = \frac{230\ V_b}{(\rho - 1)} \quad . \tag{A-45}$$

Dividing through by w gives

$$\frac{c}{w} = \frac{230}{(\rho - 1)} \ \frac{V_b}{w} \quad . \tag{A-46}$$

Setting

$$\Delta = \frac{V_b}{w}\ 10^3 \tag{A-47}$$

gives

$$\frac{c}{w} = \frac{0.230}{(\rho - 1)}\ \Delta \quad , \tag{A-48}$$

where c/w is a coordinate of the right side of figure A-3.

For between-caliber comparisons, it follows from relations A-14, A-24, A-25, A-43, and A-47 that

$$\frac{c_1/w_1}{c_2/w_2} = \frac{\Delta_1}{\Delta_2} = \frac{d_2}{d_1} \quad , \tag{A-49}$$

while for within-caliber comparisons, it follows from relations A-36, A-37, A-43, and A-47 that

$$\frac{c_1/w_1}{c_2/w_2} = \frac{\Delta_1}{\Delta_2} = \frac{w_2}{w_1} \quad . \tag{A-50}$$

Equation A-48 can be plotted with Δ as a parameter on the (ρ, c/w) space (the right side) of figure A-3. However, the resulting figure is extremely cluttered and difficult to read. Fortunately, if c/w and Δ are known, ρ is not required. Therefore, after plotting equation A-48 on the right side of figure A-3, the curves for ρ were removed to produce a nomograph whose right side employs coordinates c/w and Δ.

The resulting nomograph will be referred to as the Full-Case Nomograph. It is shown in figure A-4. This nomograph will be used to develop case capacity (V_c) categories for homologous cases. As an example of how the Full-Case Nomograph can be used, suppose a rifle of caliber d and bore length ℓ is to fire a bullet of weight w at a muzzle velocity v_o, with a peak chamber pressure $\hat{p}$, and it is desired to estimate the case volume V_c required to just hold the idealized powder. The fact that $\hat{p}$ and v_o are known determines a point on the left side of figure A-4. A horizontal line (straight edge) through ($\hat{p}$, v_o) intersects the appropriate Δ curve on the right side of the figure, where Δ is calculated from formulas A-4 and A-47. This intersection point determines c/w and, since w is known, c can be calculated from

$$c = w\,(c/w) \quad . \qquad \text{(A-51)}$$

Having found c, the case volume V_c can be calculated from formula A-43:

$$V_c = \frac{c}{230} \quad . \qquad \text{(A-52)}$$

Loadings where the powder just fills the case volume are considered to give best results, assuming the desired values of other parameters are obtained (reference A-1, page 47). The Full-Case Nomograph is dealing with an idealized situation, but the resulting case volume is acceptable as a first approximation. Furthermore, for purposes of this paper, the Full-Case Nomograph is used to arrive at categories of homologous case volumes.

For between-caliber comparisons, E_o and V_c are both proportional to d^2. Therefore, if cases are proportionally filled with powder ($c \propto V_c$), the ballistic efficiency (ϵ) is constant.

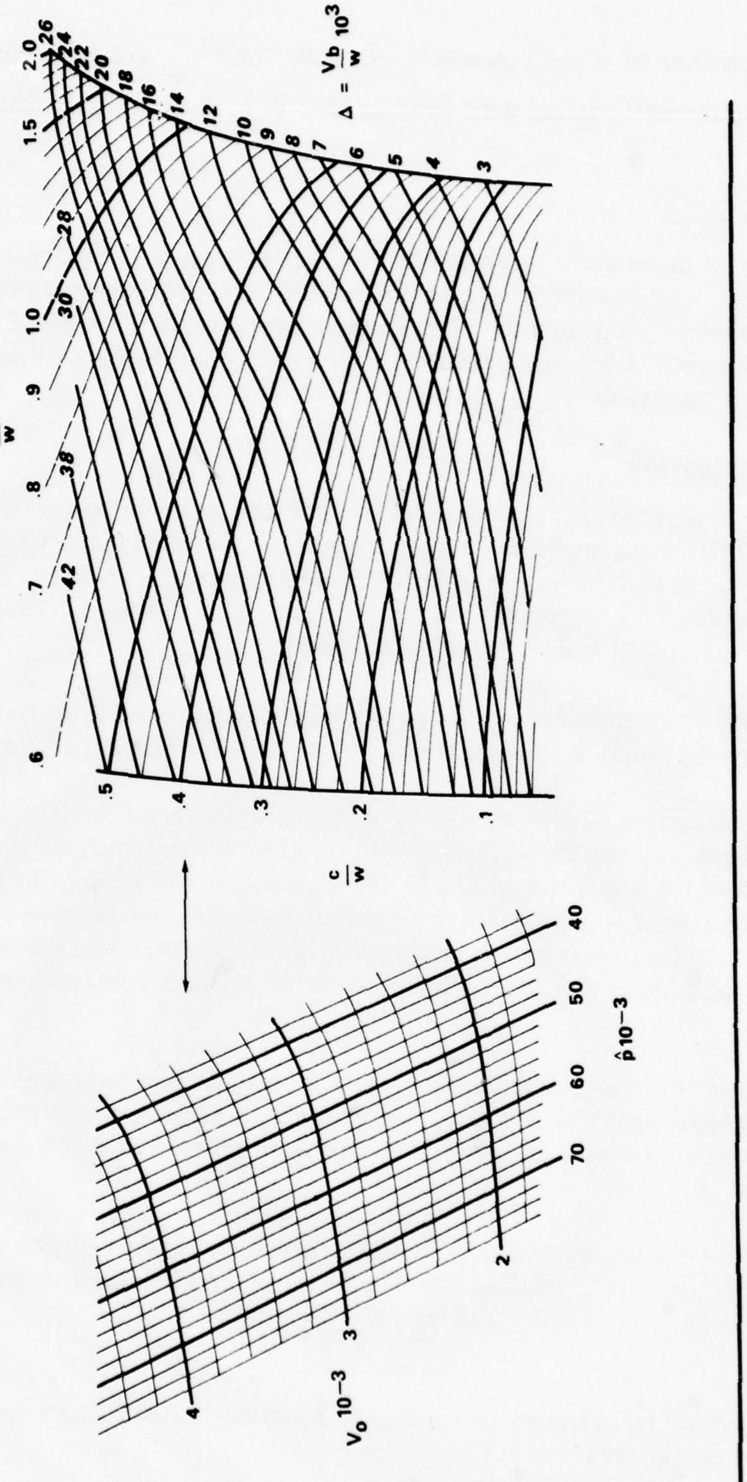

FIG. A-4: FULL-CASE NOMOGRAPH

For within-caliber (d fixed) comparisons, E_o and V_c are both constant. Therefore, if cases are proportionally filled with powder ($c \propto V_c$), the ballistic efficiency (ϵ) is again constant.

NUMERICAL VALUES

This section is devoted to establishing five bullet-weight categories and five case-volume categories, and assigning numerical values to these categories such that the homologous parameter relations developed above (under COMPARISONS INVOLVING TWO CALIBERS) hold. Additional conditions on these categories will be introduced as their development progresses.

Bullet-Weight Categories

Five bullet-weight categories[1] are defined for comparison purposes. These are: Typical Heavy (TH), Medium/Heavy (M/H), Typical Medium (TM), Medium/Light (M/L), and Typical Light (TL). The range of bullet weights from TH to TL is to roughly cover the weights of bullets commercially available for the five calibers (d =.224, .243, .264, .284, and .308) discussed under INTRODUCTION.

The difference in weights (Δ w) for two adjacent categories is to be constant for each caliber. For between-caliber comparisons, Δ w values must satisfy condition A-14.

The weight categories are obtained by taking the weight of the TM bullet for d = .284 inch as W = 136 grains, and the difference in weights for this caliber as Δ w = 18 grains. The parameter relations developed above now determine the remaining 24 bullet weights (four more for caliber d = .284, and five category weights for each of the remaining four calibers). The results are shown in table A-1, where the weights are given to four figures to permit checking of parameter relations involving muzzle velocities normally given to four figures.

Within each of the five bullet categories, the weights are homologous -- in other words, satisfy condition A-14. Hence, the weights of table A-1 are homologous by rows. Δ w values also satisfy condition A-14. Within each column of table A-1, the adjacent weights differ by Δ w.

Figure A-5 shows the homologous (within category) bullet weights from table A-1. This figure provides a quick means for determining homologous (corresponding) bullet weights for calibers from 22 (d = .224) to 30 (d = .308).

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[1] The term bullet-weight category is applied to a collection of bullets satisfying the condition of equation A-14.

TABLE A-1

HOMOLOGOUS w FOR BULLET CATEGORIES

Bullet category \ d	.224	.243	.264	.284	.308
TH	84.40	107.7	138.2	172.0	219.4
M/H	75.56	96.47	123.7	154.0	196.4
TM	66.73	85.19	109.2	136.0	173.5
M/L	57.90	73.92	94.78	118.0	150.5
TL	49.07	62.64	80.33	100.0	127.6
Δ w	8.832	11.275	14.46	18.00	22.96

T = typical, H = heavy, M = medium, L = light

Since Δ w is constant within each caliber, the five bullet-weight categories can be evenly spaced on a horizontal axis. Then, using table A-1, w/d^3 can be plotted as a function of weight category. The result, which holds for all five calibers, is shown in figure A-6.

Ballistic Coefficients

Up to this point, we have defined five categories of homologous bullet weights. The numerical values of the ballistic coefficients which go with the homologous weights must now be determined. It was decided to base this determination on some bullet which had a clean aerodynamic design as reflected by a relatively high value of C for its caliber and weight. Thus, a sort of upper bound for homologous ballistic coefficient values would be established. However, before determining homologous ballistic coefficient values to go with the five sets of homologous weights (w), some background information related to bullet design will be presented.

The ballistic coefficient (C) is a measure of a bullet's capability to maintain its velocity in air. A large ballistic coefficient represents a low coefficient of drag and is a desirable characteristic of bullets. The exterior ballistics calculations carried out for this paper employ the Ingalls Ballistic Tables and the associated ballistic coefficients. These are the same tables and coefficients employed in hand-loading manuals, such as those published by Hornady (reference A-1) and Speer (reference A-2).

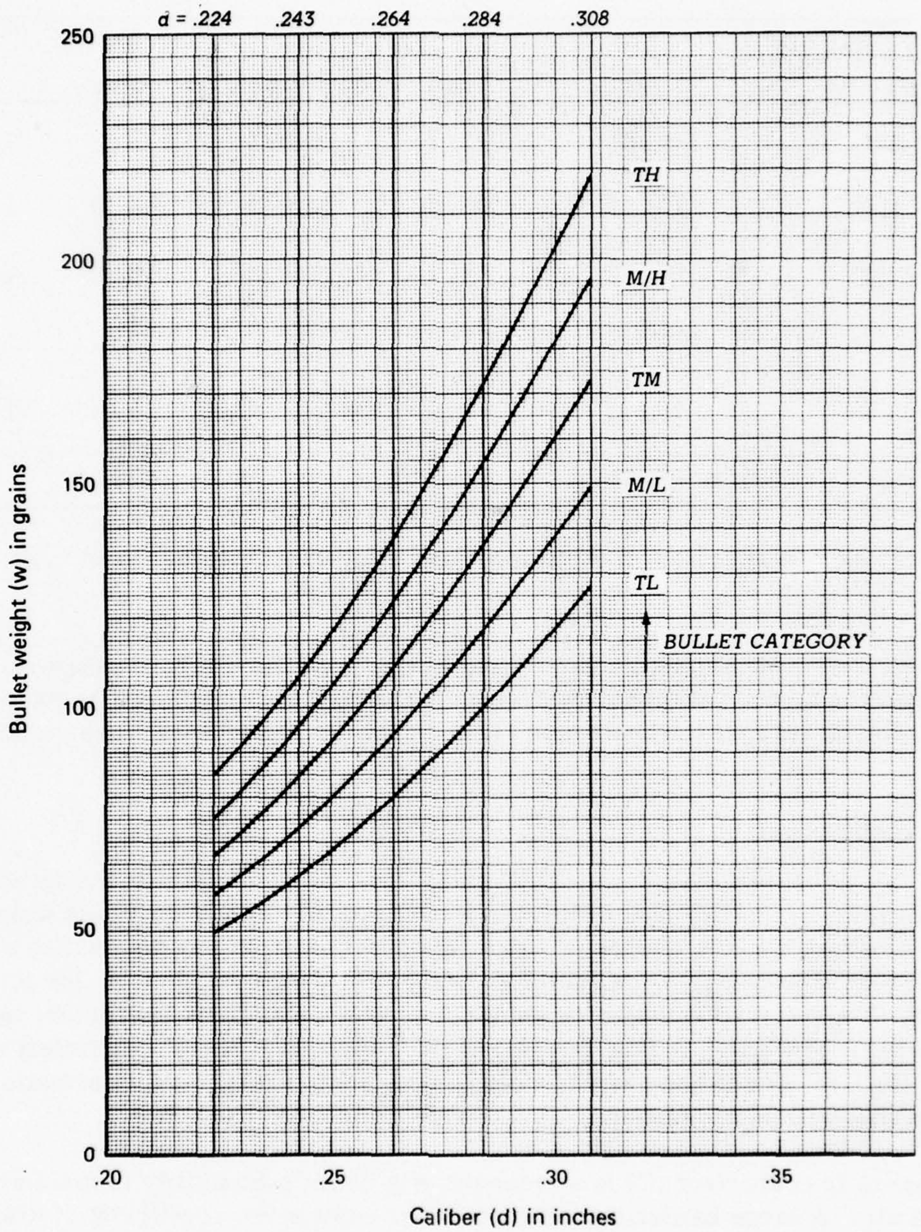

FIG. A-5: w AS A FUNCTION OF d FOR HOMOLOGOUS BULLET WEIGHT CATEGORIES

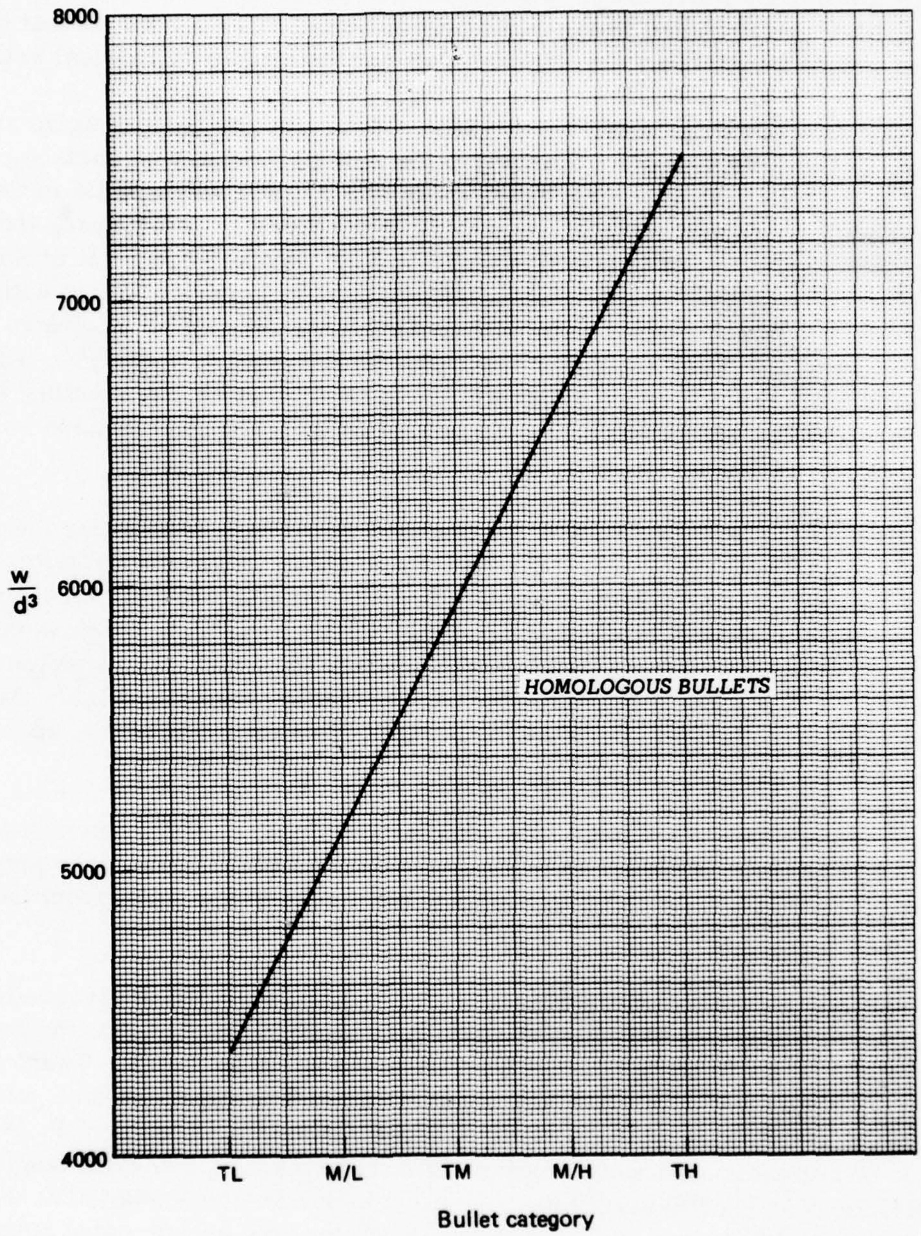

FIG. A-6: $\frac{w}{d^3}$ AS A FUNCTION OF BULLET CATEGORY FOR HOMOLOGOUS BULLET WEIGHTS

The nose and tail of a bullet appear to be the critical parts of bullet geometry so far as obtaining a low aerodynamic drag. Therefore, for purposes of this paper, all bullets of the same caliber are assumed to have identical nose and tail sections so that their differences in weight result from varying the length of the middle cylindrical section.

The two common low drag bullet nose configurations are the tangent-ogive and secant-ogive. In the case of the tangent-ogive, the cylindrical bearing surface, or body of the bullet, is tangent to the circular arc forming the bullet's nose, while in the secant-ogive, the cylindrical bearing surface is a secant to the circular arc forming the bullet's nose. These geometries are shown in figure A-7. The drawing on the left of this figure shows a tangent-ogive nose bullet, where the curve TP is the arc of a circle with center at O and having a radius OT equal to 3 calibers (3d). The line BT is tangent to the ogive circle at point T. The drawing on the right shows a secant-ogive nose bullet, where the curve SP is the arc of a circle with center at Q and having a radius QS equal to 16 calibers. The line BS is secant to the ogive circle. The head or nose lengths or heights are denoted by h.

The two bullets shown in figure A-7 represent equal overall lengths for the tangent- and secant-ogive designs. In general, the comparison of aerodynamic drag for different nose forms is not straightforward because a change in the nose shape also changes some other characteristic of the bullet. It is frequently held that the best comparison is on the basis of equal head or nose lengths which, according to The American Rifleman (reference A-3, page 68), is the way the comparison is usually done. Also, the weights should be the same, as well as the bullet base or tail design. It is frequently stated that of these two designs, the secant-ogive has less drag.

If one examines either a tangent- or secant-ogive bullet, it can be seen that the tips of the bullets are blunted slightly. This blunt tip is called the meplat and is present because it is neither desirable nor possible to bring the tip to the sharp point that the geometry dictates.

In general, any blunting of the bullet tip increases the drag and decreases the ballistic coefficient. Also, according to The American Rifleman (reference A-3), the meplat decreases the difference in drag between the two basic low drag forms. Thus, a secant-ogive design does not necessarily produce a bullet of minimal drag. In fact, of the three 180-grain 30-06 bullets tested for the National Rifle Association (by the H.P. White Laboratory), one of which was secant-ogive while the remaining two were tangent-ogive, the mean velocity loss (to 300 yards) for the secant-ogive design was greater than for either tangent-ogive (reference A-3, page 69). However, the secant-ogive bullet had a flat base and rounded nose tip, while both tangent-ogive bullets had rather sharp points and one had a boattail. Thus, the implications of the test are not clear.

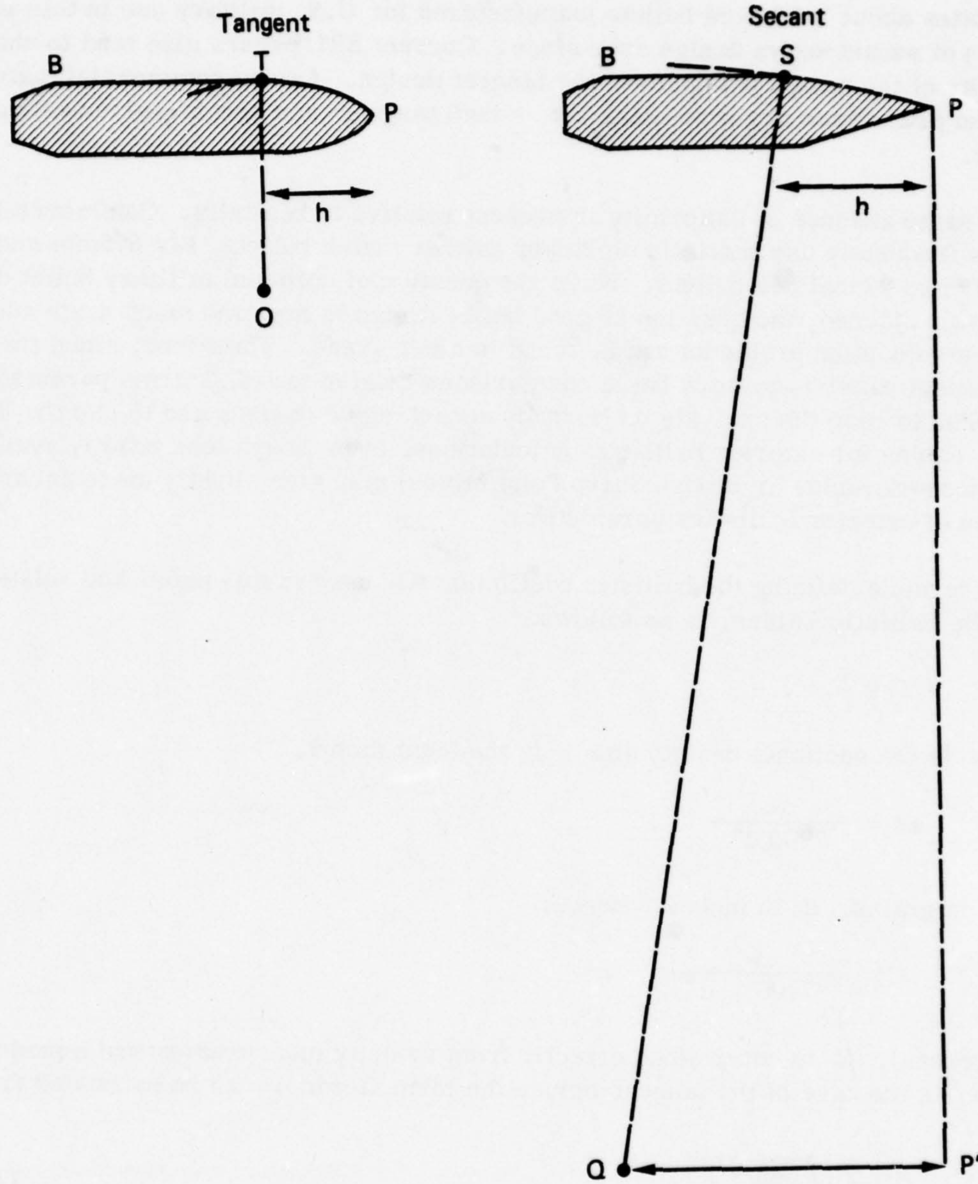

FIG. A-7: OGIVE GEOMETRIES

The secant-ogive design was standardized on the 7.62-mm. NATO round made in the United States about 1950, and bullets manufactured for U.S. military use in this round have been of secant-ogive design ever since. Current BRL papers also tend to show the superiority of the secant design over the tangent design. Among commercial match bullets the preference is not so clear cut -- both tangent- and secant-ogive designs are available.

The same absence of uniformity is present relative to boattails. Commercial manufacturers frequently use boattails on larger caliber match bullets, say 6.5mm and up, but not on 17- and 22-caliber bullets. While the question of optimum military bullet design is somewhat clouded, the question of good bullet design is not, and much more acute small-arms decision problems can be found in other areas. Therefore, since the purpose of the present paper is to draw basic comparisons related to small-arms parameters, it was decided to base the analysis on Hornady secant-ogive designs and to use the Ingalls Ballistic Tables for exterior ballistics calculations, even though less readily available tables (Hodsock tables or British Sharp Point tables) may give slightly more accurate estimates of exterior ballistics parameters.

The formula defining the ballistic coefficient C, used in this paper and related to the Ingalls Ballistic Tables, is as follows:

$$C = \frac{s}{i} \quad ,$$

where s is the sectional density and i is the form factor.

$$s = \frac{w}{7000\ d^2} \quad ,$$

with w in grains, d in inches. Hence,

$$C = \frac{w}{7000\ i\ d^2} \quad .$$

In general, C is determined directly from velocity measurements of a projectile. However, in the case of the tangent-ogive, the form factor i can be estimated from the formula,

$$i = \frac{2}{n}\sqrt{\frac{4n - 1}{7}} \quad , \qquad \text{(A-53)}$$

where n is the ogive radius measured in calibers (see reference A-5). Commerical Spitzer bullets (which have a tangent-ogive) usually have ogive radii in the neighborhood of 6 calibers.

While it should be possible to determine a formula similar to A-53 for secant-ogive bullets, none was found in the literature, nor were authorities contacted on the subject familiar with such a formula. The problem of determining a formula for the secant-ogive is complicated by the fact that two variables (the radius QS and the distance QP') are involved.

Since the ballistic coefficient for a particular caliber and bullet weight can vary widely with bullet design, it was decided to base C values on the ballistic coefficient of the Hornady 7-mm. (d=.284), 162-grain, match bullet, which has C = .725, and to consider this as an upper bound ($\hat{C}$) for determining C values. The ballistic coefficients of many commercially available bullets, as well as the M14 and M16 bullets, are considerably below what is obtained by scaling $\hat{C} = .725$ for d = .284, w = 162. In fact, they are roughly three-fourths the $\hat{C}$ values.

Hence, an additional set of ballistic coefficients, denoted by $\bar{C}$, where

$$\bar{C} = .75\ \hat{C} \tag{A-54}$$

was calculated.

The $\hat{C}$ values represent high or upper bound values, while $\bar{C}$ values are more nearly what might be expected in line with current commercial and military bullet designs.

Table A-2 shows the $\hat{C}$ and $\bar{C}$ values for each weight w. The C values by rows (homologous values) satisfy condition A-28, while the C values by columns (within-caliber) satisfy condition A-38.

Bullet designs such as the 7-mm., 162-grain, boattail, secant-ogive nose may have extremely short bearing surfaces in their lighter weights. The bearing surfaces for weight categories M/L and TL are less than 1 caliber in length. Thus, a boattail combined with a long-nose section, while yielding a high ballistic coefficient, can reduce the length of the bearing surface below an acceptable figure. When this happens, the first design feature to be sacrificed is probably the boattail. If this does not provide an acceptable bearing surface length, the nose length can be decreased. Each of these modifications, while keeping the bullet weight fixed, leads to a decrease in the ballistic coefficient. However, the homologous relations for the weight categories involved are still maintained but the within-caliber relation (A-38) no longer holds for an (entire) table, such as A-2.

TABLE A-2

BALLISTIC COEFFICIENTS

Category \ d	Parameter	.224	.243	.264	.284	.308
TH	w	84.40	107.7	138.2	172.0	219.4
	$\hat{C}$	.607	.658	.715	.769	.834
	$\bar{C}$	.455	.493	.536	.577	.625
M/H	w	75.56	96.47	123.7	154.0	196.4
	$\hat{C}$	.543	.589	.640	.688	.747
	$\bar{C}$	.407	.442	.480	.516	.560
TM	w	66.73	85.19	109.2	136.0	173.5
	$\hat{C}$	.480	.520	.565	.608	.660
	$\bar{C}$	.360	.390	.424	.456	.495
M/L	w	57.90	73.92	94.78	118.0	150.5
	$\hat{C}$	.416	.451	.490	.528	.572
	$\bar{C}$	.312	.339	.368	.396	.429
TL	w	49.07	62.64	80.33	100.0	127.6
	$\hat{C}$	.353	.383	.416	.447	.485
	$\bar{C}$	.264	.287	.312	.335	.364

At the heavy end of the bullet range, designs such as the Hornady (secant-ogive boat-tail) may lead to bullets which are excessive in length. This condition can present numerous problems with seating depth, space available for powder, and chambering the round in full automatic fire. Thus, it is advisable in carrying out design analysis to give perference to bullet weights in the range M/L to M/H, rather than those at the extremes (TL or TH) of the weight range. Historically, the United States military rifles have fired bullets in the M/L to M/H weight range. The recently proposed 6-mm. SAW round (XM732) is a departure from the use of M/L to M/H bullets with comparable case sizes. This round employed a 105-grain bullet which belongs (approximately) to the TH category.

Case Volume Categories

Five case-volume categories[1] are defined for comparison purposes. These are: Typical Large (TL), Medium/Large (M/L), Typical Medium (TM), Medium/Small (M/S), and Typical Small (TS).

---

[1] The term case-volume category is applied to a collection of cases satisfying the condition of equation A-25.

As was the situation with bullet-weight categories, the range of case volumes from TL to TS is to roughly cover case volumes commercially available for the five calibers (d = .224, .243, .264, .284, and .308). However, instead of defining these case categories in an independent fashion, as was done for bullet weights, the case-volume categories will be defined in relation to the bullet-weight categories, using the Full-Case Nomograph (figure A-4).

Before developing the V_c categories, some background discussion is in order. First of all, bullets are available in various geometric configurations, such as round nose, semi-wadcutter, secant-ogive, and tangent-ogive, and in a range of weights for each caliber. A bullet of given caliber (d) may be loaded in several different cases whose volumes may vary considerably. However, heavy bullets of a given caliber tend to be loaded in large cases, while light bullets tend to be loaded in small cases.

The situation for cases is quite different. Cases, except for new developments, must conform to specific rifle chambers which are designated by the round name. The capacities of cases (in cubic inches) for some well-known rounds are shown in table A-3. These case capacities were taken from reference A-4.

To define the five case-volume categories and assign numerical values to their volumes, the following requirements are laid down.

The homologous relations for between-caliber comparisons hold for case-bore parameters.

The length of the bore will be fixed at ℓ = 22 inches, for all calibers.

When the five case-volume categories are loaded with corresponding bullet weights in caliber d = .284 and fired in a bore having ℓ = 22 inches at peak chamber pressure $\hat{p}$ = 50,000 psi, the resulting muzzle velocities are to be constant at v_o = 2800 feet per second (fps).

Table A-4 shows the pertinent parameter values for calculating V_c values of each case category of caliber d = .284 inches. Values of w are taken from table A-1. Values of Δ are calculated from formula A-47, and values of ρ are calculated from formula A-16.

The Full-Case Nomograph (figure A-4) is used to estimate c/w for $\hat{p}$ = 50,000, v_o = 2800 fps, and the calculated value of Δ. Values for c and V_c are calculated from formulas A-51 and A-52, respectively.

TABLE A-3

CAPACITIES OF POPULAR CARTRIDGES[a]

Capacities are measured to the base of a normally seated bullet in the instance of straight cases, and to the junction of neck and shoulder in bottle neck cases. Cubic capacity is determined by weighing amount of water case will hold, and calculating from that the volume in cubic inches.

Case	V_c (cu. in.)	Case	V_c (cu. in.)
.22 Hornet (late)	.045	.270 Winchester	.250
.22 K-Hornet	.053	.280 Remington	.245
.218 Bee	.059	.284 Winchester	.247
.222 Remington	.094	7mm Mauser	.211
.222 Rem. Mag.	.114	7mm Rem. Mag.	.317
.223 Remington	.112	.30 Carbine	.059
.219 Wasp	.107	.30/30 Winchester	.142
.219 Zipper	.131	.30 Remington	.147
.224 Weatherby	.142	.300 Savage	.184
.225 Winchester	.151	.30/40 Krag	.188
.22/250 Remington	.167	.308 Winchester	.198
.220 Swift	.177	.30/06 Springfield	.242
.243 Winchester	.200	.300 H & H Mag.	.318
6mm Remington	.204	.308 Norma Mag.	.322
.25/20 Winchester	.058	.300 Win. Mag.	.332
.25/35 Winchester	.134	.300 Weatherby	.364
.25 Remington	.140		
.250 Savage	.166		
257 Roberts	.213		
6.5mm Jap.	.174		
6.5mm M-S	.179		
6.5x55mm	.206		
6.5mm Rem. Mag.	.260		
.264 Win. Mag.	.318		

[a] Courtesy of Outdoor Life/Popular Science Books.

TABLE A-4

CASE VOLUME CATEGORIES/NUMERICAL VALUES FOR
$d = .284$ inches, $\ell = 22$ inches, $V_b = 1.394$ cu.in., $\hat{p} = 50,000$ psi, $v_o = 2800$ fps

Bullet category	w	Δ	Case category	c/w	c	V_c	ρ
TH	172	8.1025	TL	.359	61.75	.2685	6.192
M/H	154	9.0496	M/L	.344	52.98	.2303	7.053
TM	136	10.247	TM	.325	44.20	.1922	8.253
M/L	118	11.810	M/S	.300	35.40	.1539	10.06
TL	100	13.936	TS	.266	26.60	.1156	13.06

It should be mentioned that while there is a general tendency to employ bullets and cases which are "matched in size," the above calculations are merely an exercise to classify case volumes associated with a given caliber. Any one of the five case categories can be loaded with any one of the five bullet categories -- though the results of some of these combinations may leave much to be desired.

Just as was the case for bullet-weight categories, the case-volume categories have to satisfy the homology relation (equation A-25). The results are shown in table A-5 where V_c is given to four decimals for purposes of checking.

Figure A-8 shows the data from table A-5 in graphical form and figure A-9 shows V_c/d^3 as a function of case category.

Next, we shall examine a partial set of parameter values when corresponding bullet and case categories are used in forming the rounds. This is an extension of the data contained in tables A-4 and A-5 and provides an opportunity to check the mean effective ($\tilde{p}$) and peak chamber ($\hat{p}$) pressure values. The values of $\tilde{p}$ were calculated using formula A-42, while values of $\hat{p}$ were estimated using the Full-Case Nomograph (figure A-4). It is to be noted that the value of $\tilde{p}$ is constant within rows -- thereby satisfying the homology requirement for between-caliber comparisons. It appears from table A-6 that the peak chamber pressure is roughly constant at 50,000 psi. The reader should note that table A-6 does not satisfy the condition (V_c constant) for within-caliber comparisons. Therefore, within-caliber E_o and $\tilde{p}$ values are not constant but velocities are.

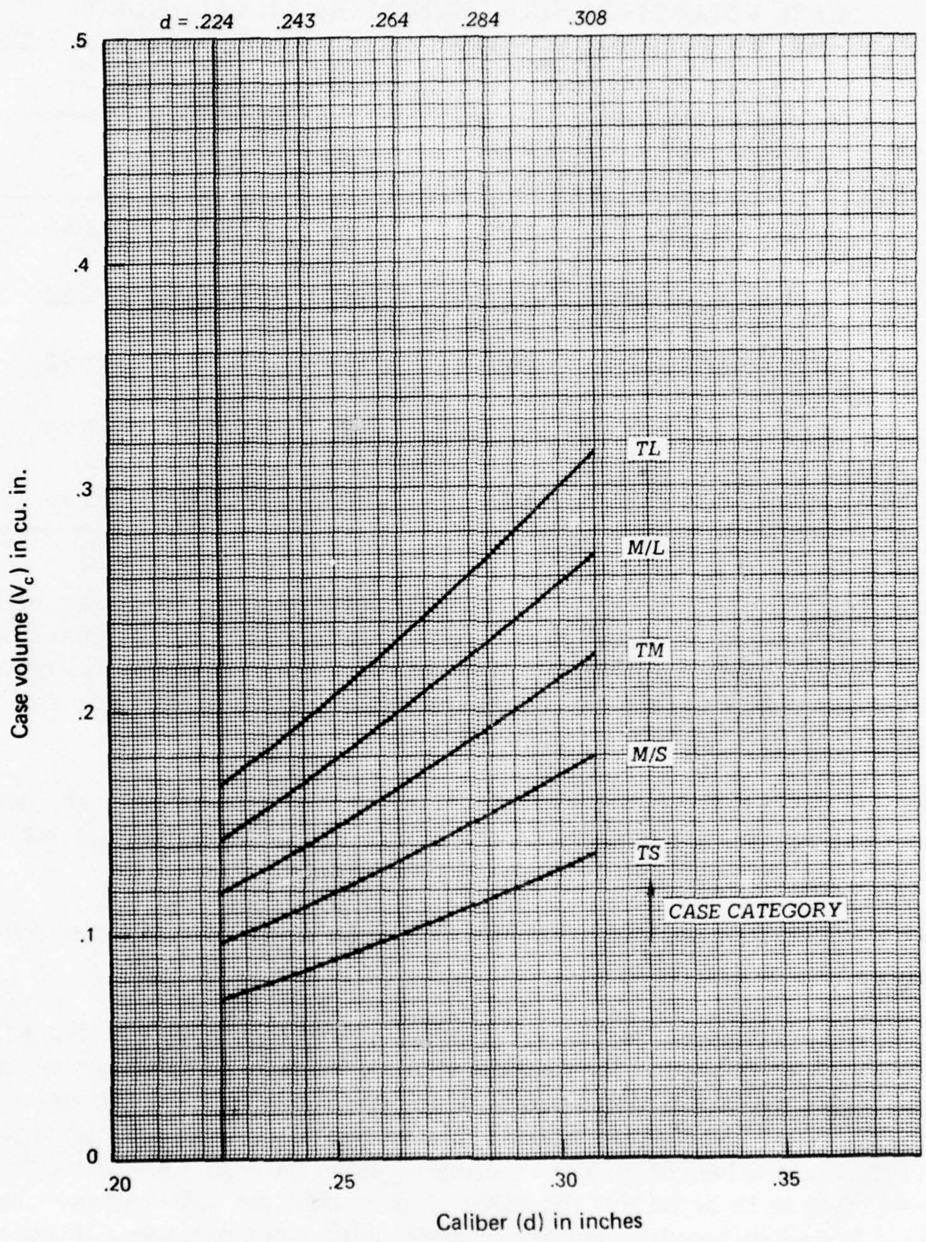

FIG. A-8: CASE VOLUME CATEGORIES

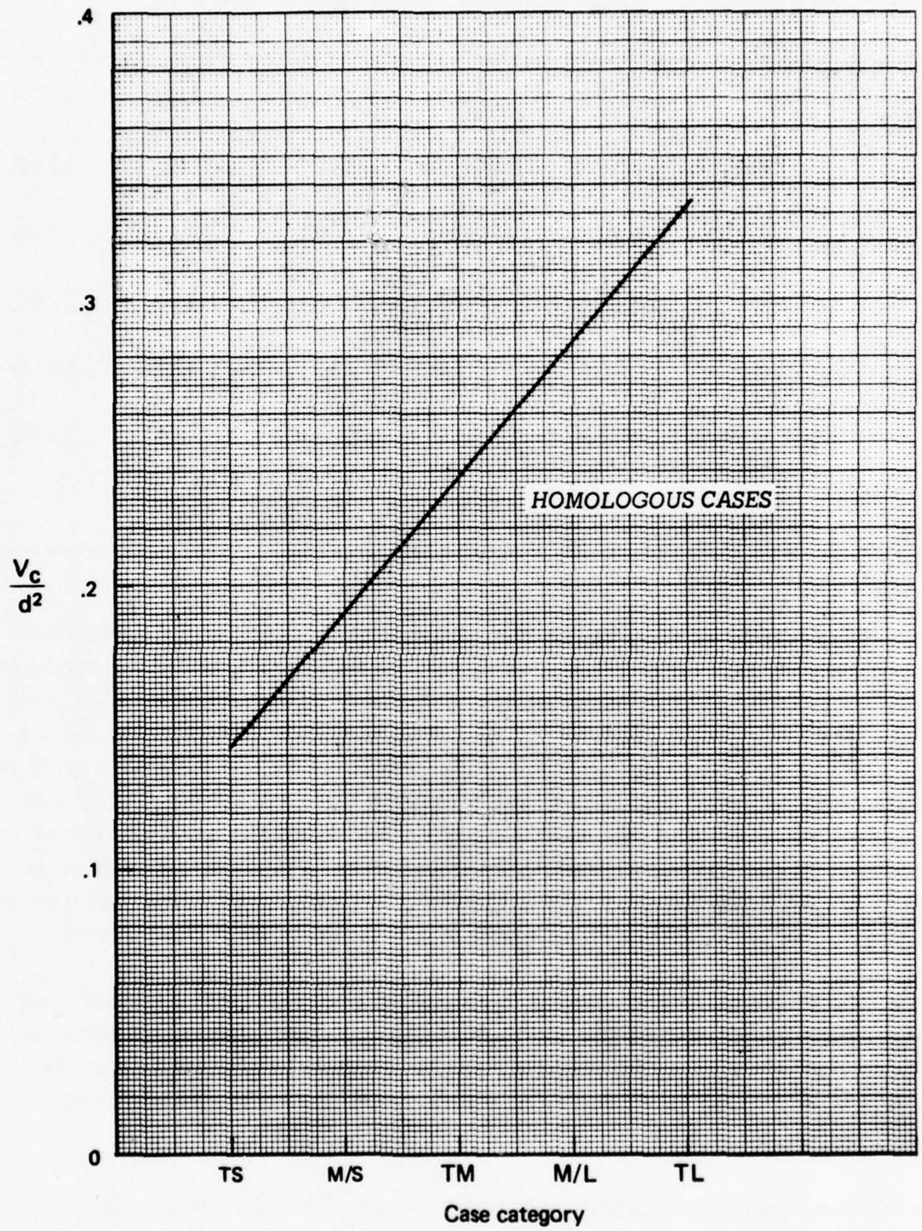

FIG. A-9: $\frac{V_c}{d^2}$ AS A FUNCTION OF CASE CATEGORY FOR HOMOLOGOUS CASE VOLUMES

TABLE A-5

CASE VOLUME CATEGORIES, NUMERCIAL VALUES V_c

Case category	d / ρ	.224	.243	.264	.284	.308
TL	6.192	.1670	.1966	.2320	.2685	.3158
M/L	7.053	.1433	.1686	.1990	.2303	.2709
TM	8.253	.1196	.1407	.1661	.1922	.2261
M/S	10.06	.0957	.1127	.1330	.1539	.1810
TS	13.06	.0719	.0846	.0999	.1156	.1360
		ρ values are for ℓ = 22				

The primary purpose in developing homologous case-volume categories and defining their numerical values is to provide a means for comparing the sizes of cases of two calibers. Furthermore, each category (row of table A-5) provides a set of homologous case volumes. However, this analysis is based on idealized powder and, if a set of homologous cases is to be loaded with available powders, there may very well be some combinations of case volume and bullet weight where the case will not hold sufficient powder to provide the required muzzle velocity. One solution to this problem is to use a set of homologous cases having slightly larger volumes. In doing this, it is not necessary to use the next larger case category (of table A-5) but simply to construct a new case category (using formula A-25) which is sufficiently large to hold the powder.

However, the important parameters involved in between- and within-caliber comparisons are those associated with bullet weights, ballistic coefficients, muzzle velocities, and pressures, rather than case volumes. If all of these important parameters meet the requirements, it should be possible to resolve the case-volume problem, if it arises, with real powders.

It is interesting that the recently proposed 6-mm. SAW round (XM732) not only employed a 105-grain bullet but it utilized the M16 round case, having $V_c \simeq .112$ cubic inches. Therefore, the round combined (approximately) a TH bullet in a M/S case. It appears that no commercially available rifle round, employing a similar combination of bullet weight and case volume, exists. Apparently, such combinations are quite unusual.

TABLE A-6

PARAMETER VALUES FOR CORRESPONDING BULLET CASE CATEGORIES

(ℓ = 22)

Category w	V_c						
		d	.224	.243	.264	.284	.308
		v_o	3153	3027	2804	2800	2689
		V_b	.8670	1.020	1.204	1.394	1.639
TH		w	84.40	107.7	138.2	172.0	219.4
		E_o	1864	2193	2589	2995	3524
		$10^{-6}\gamma$	25.79	25.78	25.79	25.79	25.79
	TL	V_c	.1670	.1966	.2320	.2685	.3158
		c/w	.4551	.4199	.3861	.3590	.3311
		ρ	6.19	6.19	6.19	6.19	6.19
		Δ	10.27	9.47	8.71	8.10	7.47
		$10^{-6}\beta$	50	50	50	50	50
M/H		w	75.56	96.47	123.7	154.0	196.4
		E_o	1668	1963	2317	2682	3154
		$10^{-6}\gamma$	23.09	23.09	23.09	23.09	23.09
	M/L	V_c	.1433	.1686	.1990	.2303	.2709
		c/w	.4362	.4020	.3700	.3440	.3172
		ρ	7.05	7.05	7.05	7.05	7.05
		Δ	11.47	10.58	9.74	9.05	8.35
		$10^{-6}\beta$	50	50	50	50	50
TM		w	66.73	85.19	109.2	136.0	173.5
		E_o	1473	1734	2045	2368	2786
		$10^{-6}\gamma$	20.39	20.39	20.39	20.39	20.40
	TM	V_c	.1196	.1407	.1661	.1922	.2261
		c/w	.4122	.3799	.3498	.3250	.2997
		ρ	8.25	8.25	8.25	8.25	8.25
		Δ	12.99	11.98	11.03	10.25	9.45
		$10^{-6}\beta$	50	50	50	50	50
M/L		w	57.90	73.92	94.78	118.0	150.5
		E_o	1278	1504	1775	2055	2417
		$10^{-6}\gamma$	17.69	17.69	17.69	17.69	17.69
	M/S	V_c	.0957	.1127	.1330	.1539	.1810
		c/w	.3802	.3507	.3227	.3000	.2766
		ρ	10.06	10.05	10.05	10.06	10.06
		Δ	14.97	13.80	12.71	11.81	10.89
		$10^{-6}\beta$	50	50	50	50	50
TL		w	49.07	62.64	80.33	100.0	127.6
		E_o	1083	1275	1505	1741	2049
		$10^{-6}\gamma$	14.99	14.99	14.99	14.99	15.00
	TS	V_c	.0719	.0846	.0999	.1156	.1360
		c/w	.3370	.3106	.2860	.2659	.2451
		ρ	13.06	13.06	13.05	13.06	13.05
		Δ	17.67	16.29	14.99	13.94	12.85
		$10^{-6}\beta$	50	50	50	50	50

We next proceed to develop comparison tables, which are similar to table A-6 but satisfy both between and within-caliber parameter relations. Such tables provide the bases for constructing solution spaces satisfying various sets of military small-arms requirements.

COMPARISON TABLES

The term comparison table is used to denote a table of parameter values satisfying both between- and within-caliber relations. Thus, the within-caliber case volume is constant. Comparison tables are constructed by selecting a single case category and combining it with all five bullet categories. Therefore, one of the rows of table A-6 will appear in each comparison table -- if the ℓ and $\hat{p}$ values are held at 22 inches and 50,000 psi, respectively.

The following comparison table (table A-7) employs a TM case with all five bullet categories. Thus, the third category combination (TM, TM) from table A-6 appears as the third category combination in the comparison table. The remainder of the comparison table can be calculated from this row, using within-caliber relations. The rows of table A-7 satisfy the between-caliber (homology) relations, while the columns satisfy the within-caliber relations.

Peak chamber pressures ($\hat{p}$) for table A-7 were checked using figures A-3 and A-4. *Considering the difficulty of reading* values from the nomographs, all $\hat{p}$ values appear to be approximately 50,000 psi.

In general, the development of parameter values in this appendix is based on a peak chamber pressure $\hat{p} \simeq 50{,}000$ psi. However, estimates of case volumes (V_c) associated with peak chamber pressures of 52,000 and 48,000 psi are also of interest, and these pressures are used in the development of solution spaces presented in the main text.

For between-caliber (homologous rounds) comparisons, the case volume is proportional to the square of the caliber (formula A-25), while for within-caliber comparisons the case volume is constant (formula A-37). Therefore, to estimate the set of homologous case volumes associated with a given value of $\hat{p}$, all that is required is the case volume for one bullet category of one caliber associated with the desired muzzle velocity (v_o) and peak chamber pressure ($\hat{p}$). Thus, $\hat{p}$ and v_o are known, Δ is calculated from formula A-47 (assuming $\ell = 22$ inches). Then, assuming full cases (idealized powder) the value of c/w can be read from figure A-4 and the case-volume estimate calculated from formula A-43. Homologous case volumes for other calibers are then calculated using formula A-25.

TABLE A-7

COMPARISON TABLE FOR TYPICAL MEDIUM CASES

($\hat{p}$ = 50,000, $\tilde{p}$ = 20,390)

Bullet Category						
	d	.224	.243	.264	.284	.308
	V_c=TM	.1196	.1407	.1661	.1922	.2261
	V_b	.8670	1.020	1.204	1.394	1.639
	ρ	8.25	8.25	8.25	8.25	8.25
TH	w	84.40	107.7	138.2	172.0	219.4
	v_o	2804	2692	2582	2490	2391
	E_o	1474	1734	2046	2369	2786
	$\tilde{p}$	20,400	20,390	20,390	20,400	20,400
	c/w	.3259	.3004	.2764	.2570	.2370
	Δ	10.27	9.471	8.712	8.105	7.470
M/H	w	75.56	96.47	123.7	154.0	196.4
	v_o	2962	2844	2729	2631	2527
	E_o	1472	1733	2046	2368	2786
	$\tilde{p}$	20,380	20,380	20,390	20,390	20,400
	c/w	.3640	.3354	.3088	.2870	.2647
	Δ	11.47	10.58	9.73	9.052	8.345
TM	w	66.73	85.19	109.2	136.0	173.5
	v_o	3153	3027	2904	2800	2689
	E_o	1473	1734	2045	2368	2786
	$\tilde{p}$	20,390	20,390	20,390	20,390	20,400
	c/w	.4122	.3799	.3498	.3250	.2997
	Δ	12.99	11.98	11.03	10.25	9.447
M/L	w	57.90	73.92	94.78	118.0	150.5
	v_o	3385	3250	3118	3006	2887
	E_o	1474	1734	2047	2368	2786
	$\tilde{p}$	20,400	20,390	20,400	20,390	20,400
	c/w	.4750	.4377	.4030	.3746	.3455
	Δ	14.97	13.80	12.70	11.81	10.89
TL	w	49.07	62.64	80.33	100.0	127.6
	v_o	3676	3530	3386	3265	3135
	E_o	1473	1734	2046	2368	2785
	$\tilde{p}$	20,390	20,390	20,390	20,390	20,390
	c/w	.5605	.5166	.4755	.4420	.4075
	Δ	17.67	16.28	14.99	13.94	12.84

Table A-8 shows the case volumes for d = .284 inches and bullet weight w = 136 grains at $\hat{p} \simeq$ 52,000, 50,000 and 48,000 psi and the associated muzzle velocities (v_o). Mean effective pressures $\tilde{p}$ are calculated from formulas A-7 and A-42.

TABLE A-8

CASE VOLUMES (V_c) FOR .284-CALIBER, 136-GRAIN BULLET AS A FUNCTION OF PEAK CHAMBER PRESSURE

$\hat{p}$	$\tilde{p}$	d	w	v_o	Δ	c/w	V_c
52,000	23,830	.284	136	3027	10.25	.412	.244
50,000	20,390	.284	136	2800	10.25	.325	.192
48,000	17,240	.284	136	2575	10.25	.240	.142

Table A-9 shows the homologous case volumes associated with the above three peak chamber pressures for the five basic calibers employed. The associated muzzle velocities can be calculated using formula A-30 and the v_o values for d = .284 inches of table A-8. Mean effective pressures ($\tilde{p}$) are calculated from formula A-42.

TABLE A-9

CASE VOLUMES (V_c) AS A FUNCTION OF CALIBER (d) AND PEAK CHAMBER PRESSURE $\hat{p}$

$\hat{p}$	$\tilde{p}$	d =	.224	.243	.264	.284	.308
52,000	23,830		.152	.179	.211	.244	.287
50,000	20,390		.120	.141	.166	.192	.226
48,000	17,240		.088	.104	.123	.142	.167

CALIBER (d) AS A FUNCTION OF w, v_o, AND $\tilde{p}$

If homologous round values for w and v_o are plotted in the (w, v_o) plane, every point within the region bounded by $.224 \leq d \leq .308$, and $TL \leq w \leq TH$ determines the caliber (d) associated with the point (w, v_o) and a given mean effective pressure $\tilde{p}$. Furthermore, all (w, v_o) combinations for a caliber $.224 \leq d \leq .308$ lie on a curve within this region

as is indicated by figure 2 of the main text. It is frequently desirable to estimate the caliber (d) associated with a given point (w, v_o) not lying on one of the five caliber curves. For example, the upper left corner of figure 10 of the main text has w = 90 grains and v_o = 3000 fps, and it is desirable to estimate the caliber associated with this point.

Fortunately, if the constant of proportionality is determined for relation A-11, the caliber d can be calculated as a function of w, v_o, and $\tilde{p}$, where the $\tilde{p}$ values associated with $\tilde{p} \simeq$ 52,000, 50,000, and 48,000 are taken from table A-9.

The resulting equation is:

$$d = 5.8254 \times 10^{-3}\ v_o \sqrt{\frac{w}{\ell \tilde{p}}} \quad . \tag{A-55}$$

Substituting w = 90 grains, v_o = 3000 fps, and $\tilde{p}$ = 23,830 psi in A-55 yields d = .229 inches, which is the caliber associated with the upper left corner of figure 10.

ROUND WEIGHT

It is frequently desirable to have estimates of round weight to use in conjunction with data such as that contained in table A-7. An estimate of total round weight can be obtained by adding the weight of the case (w_c) to the sum of the powder weight (c) and the bullet weight (w). Thus, the estimate of round weight (w_r) is given by

$$w_r = w_c + c + w \quad , \tag{A-56}$$

where all weights are in grains and w_c is the weight of a primed case. Values of c and w can be calculated from the formulas of this appendix as was done in obtaining w and c/w for table A-7. All that remains is to obtain an estimate of the case weight (w_c). It should be mentioned that case weights can vary as much as 10 grains depending on the manufacturer. Also, the case head makes a major contribution to case weight. In general, cases from the 243 Winchester to the 300 Winchester magnum have large case heads, while 22-caliber cases, such as the 5.56-mm. and 222 Remington, have much smaller heads. Several of the larger head cases, ranging in caliber from .243 to .308, were weighed (including primers) and a least square linear fit calculated expressing (primed) case weight (w_c) as a function of case volume (V_c). The result was

$$w_c = 47.3 + 603\ V_c \quad , \tag{A-57}$$

where V_c is in cubic inches and w_c is in grains.[1]

Combining formulas A-56 and A-57, the round weight estimate for large-head cases (ranging from d = .243 to .308), is given by:

$$w_r = 47.3 + 603\, V_c + c + w, \qquad \text{(A-58)}$$

where c, w and w_r are in grains and V_c is in cubic inches.

CHANGING ONE PARAMETER

The nomographs (figures A-3, A-4) can frequently be used to estimate changes in various parameters which result from changing a single parameter, such as bore length (ℓ) or powder weight (c). Thus, such questions as, what happens to the muzzle velocity when the barrel length is changed, or what parameter changes result from changing the powder weight, can be answered. It should be noted that such changes cannot be added to a comparison table, such as table A-7, because they violate some of the basic parameter relations involved in within-caliber comparisons. Changes in such basic parameters require the calculation of a new comparison table.

Changing the Bore Length

In general, shortening the length of the barrel decreases the muzzle velocity. Various formulas or rules are available for estimating the change in muzzle velocity for a decrease (usually of one inch) in barrel length. However, in theory, the use of Figure A-3 or A-4 should be more accurate for estimating the resulting change in v_o because these nomographs involve more of the small-arms parameters than are normally contained in the formulas or rules.

As an example, consider the 6-mm. (.243 caliber) round composed of the M/L bullet (w = 73.92 grains) and TM case (V_c = .1407 cu.in.) fired from a barrel having a bore length of 22 inches at a muzzle velocity v_o = 3250 fps. Suppose the barrel length is increased to 25 inches, what is the estimate of the new muzzle velocity? First calculate the new ρ and Δ, obtaining $\rho = 9.24$ and $\Delta = 15.68$. Now, c/w = .4377 remains the same as does $\hat{p}$ = 50,000 psi (see table A-7). From figure A-3, using $\hat{p}$ = 50,000 psi, c/w = .4377 and $\rho = 9.24$, the muzzle velocity is estimated as $v_o \simeq 3325$ fps. From figure A-4, using

[1] Formula A-57 does not hold for small head cases such as the 5.56-mm. and 222 Remington. The 5.56-mm. or 223 Remington primed case has V_c = .112 cubic inches and w_c=95.5 grains.

$\hat{p}$ = 50,000 psi, c/w = .4377 and Δ = 15.68, the muzzle velocity is estimated as $v_o \sim 3325$ fps. Thus, increasing the bore (and barrel) length by 3 inches results in an increase of approximately 75 feet per second in the muzzle velocity -- from 3250 to ~3325 feet per second. Hence, firing this 6-mm. round in a rifle with ℓ = 22 inches and in a machine gun with ℓ = 25 inches (3 inches longer) would not increase the down-range velocity for the machine gun substantially. A 75-feet-per-second variation in muzzle velocity can be obtained from rounds from the same ammunition lot fired in the same weapon. The way to obtain a payoff in increased muzzle velocity is to increase the case size and propellant load, and the way to obtain a payoff in increased down-range energy is to increase the caliber.

Changing the Powder Weight

Consider again the 6-mm. (d = .243 inches, w = 73.92 grains, V_c = .1407 cu.in., and v_o = 3250 fps) round of table A-7. The powder weight for this round is c = w(c/w) = 73.92 (.4377) = 32.35 grains. Suppose we wish to increase muzzle velocity for the 73.92-grain bullet by using a larger case having V_c = .1826 which will be just filled with powder. From formula A-43, the powder weight for this new case is c = 230 (.1826) = 42.00 grains. The new powder-to-bullet weight ratio is c/w = 42.00/73.92 = .5682; Δ remains the same (Δ = 13.80), but the new expansion ratio is $\rho = 1 + 1.020/.1826 = 6.586$. Hence, the two coordinates for the right sides of the nomographs, figures A-3 and A-4, are known (c/w, ρ or c/w, Δ, respectively). The new muzzle velocity (v_o) can be obtained without the use of either nomograph, but an estimate of the new peak chamber pressure ($\hat{p}$) requires using either figure A-3 or A-4.

Use formula A-39 to obtain the new muzzle velocity when each category bullet weight is loaded in each of the five case categories, noting (from table A-6) that the muzzle velocity for d = .243 inches, with corresponding bullet case categories, is 3027 feet per second.

The resulting data can be plotted as shown in figure A-10. There is a curve for each bullet weight. The curve of interest is the one for 73.92 grains. The muzzle velocity for V_c = .1826 cu.in. on this curve is v_o = 3560 fps (each centimeter square on the horizontal axis equals .0056 cu.in.).

Using v_o = 3560 fps and the above new values for c/w and ρ or Δ, $\hat{p}$ can be estimated from figure A-3 or A-4. The estimate for peak chamber pressure is $\hat{p} \simeq 52{,}000$ psi. Hence, if the 6-mm., 73.92-grain bullet is loaded in a case having a volume of .1826 cu.in., and this case is just filled with (idealized) powder, the muzzle velocity will be about 3560 feet per second and the peak chamber pressure will be approximately 52,000 psi.

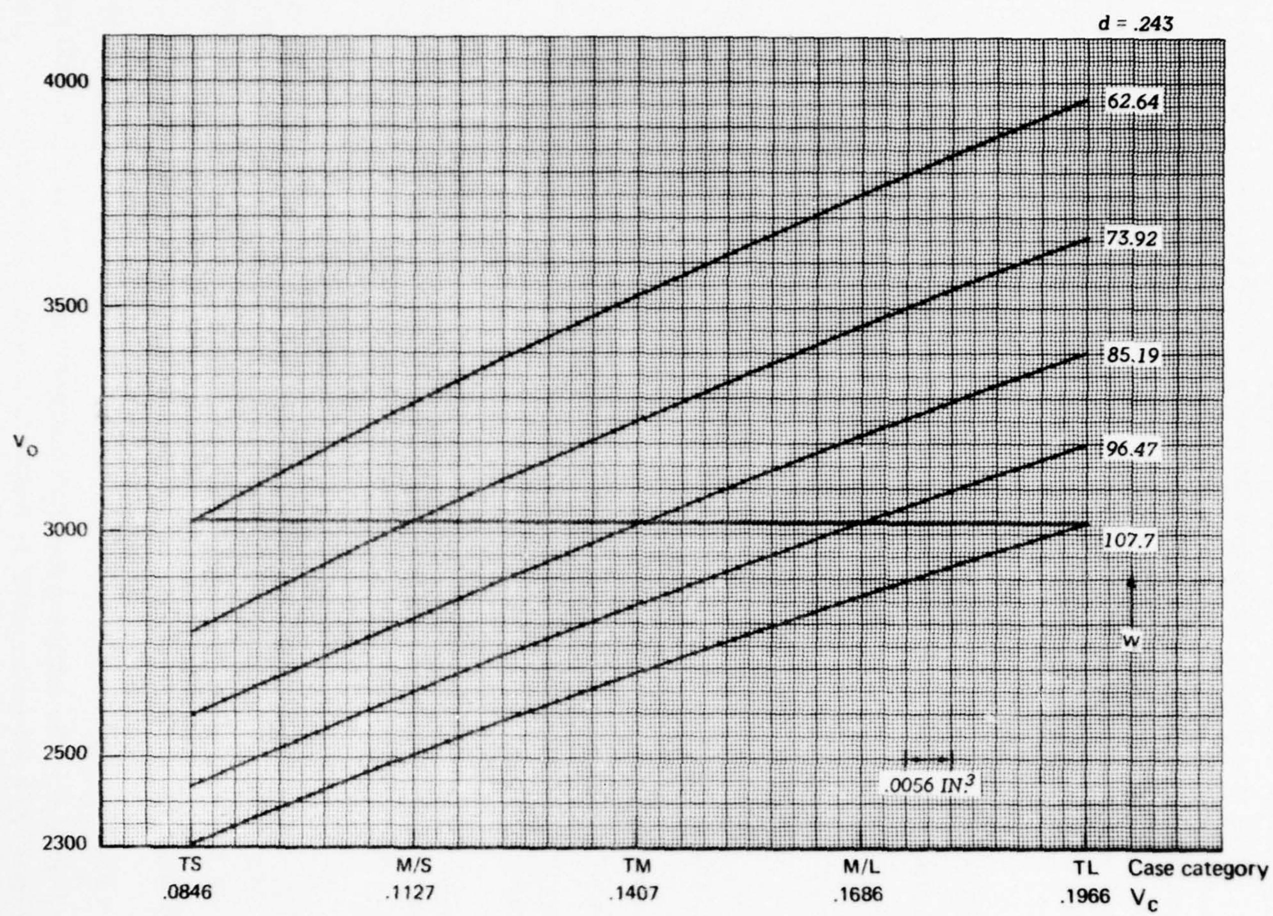

FIG. A-10: MUZZLE VELOCITY vs. CASE CAPACITY FOR GIVEN BULLET CATEGORY – .243 CALIBER

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APPENDIX B

ROUND DESIGN AND RIFLING TWIST

APPENDIX B

ROUND DESIGN AND RIFLING TWIST

This appendix discusses the components of small-arms rounds -- primer, propellant, case, and bullet -- with emphasis on the factors that influence the design of the case and bullet. Methods for estimating the rifling twist required to stabilize a bullet are also presented.

A wide variety of different primers is available which vary in size and in intensity of ignition. The intensity of ignition is determined by the priming compound's composition and weight, which can be varied at the time of manufacture. For purposes of this paper, the only requirement on primers is that they be matched to the round and provide the required ignition characteristics. This is a relatively simple matter because of the large number of primers that are available or that can be readily manufactured.

Many different propellants are available, which burn at different rates and have their own pressure characteristics when used with a particular primer, case, and bullet. The composition and physical construction of the propellant grains determines these characteristics and is beyond the scope of this research contribution. In general, for each primer, case, and bullet combination, some propellant should exist which behaves similarly to the idealized propellant associated with the TECHNIK nomograph.

CASE DESIGN

Before the advent of modern propellants, followed by the trend to smaller calibers and higher velocities, straight cartridge cases were employed. Straight cases (such as the 45-70) were tapered to ease the problem of extraction. Modern cases (except for handgun rounds) are bottlenecked because a straight case would have to be excessively long to hold the powder required to attain modern velocities. The recent trend to autoloading rifles has encouraged the development of so-called "fat cases" to provide the necessary powder capacity, while holding down the linear movement required to extract the fired case. Both straight and bottleneck rifle cases have some case taper -- the diameter of the case at the shoulder is less than at the base. Too much taper tends to place increased force on the bolt face, while too little tends to increase extraction difficulty. Both conditions are more critical with autoloading and fully automatic than with manually operated weapons.

Extraction difficulties have plagued the development of steel cases. This is frequently attributed to the (nearly) equal expansion coefficients of the chamber and case steels. Most steel cases are duplicates of cases originally designed for brass fabrication, which certainly does not help the situation.

During recent years, primers have been improved and new powders have been developed. At the same time, receiver designs have been improved and fabricated of stronger steel. Thus, it has become possible to introduce rounds which develop considerably higher peak chamber pressures than those tolerated by older rifle designs. Safety considerations dictate that these modern rounds have a geometry which will not permit their being fired (through oversight) in older rifle designs when the result would be to blow the rifle apart. This safety consideration has probably had more effect on case design than has the search for optimum case geometry as a function of bullet caliber, weight, and muzzle velocity. It is not unusual to encounter families of cases developed from a popular case by reforming it to produce a new round. For example, the 30-06 case was modified to produce the 270 Winchester and 25-06 Remington rounds.

Thus, there are several indications that optimum case design has not been a primary consideration in developing new rounds.

Various "improved rounds," retaining the original caliber, have been developed. The word "improved" applies to the case, since the caliber is retained. In general, an improved case is obtained by reforming to increase propellant capacity. This is accomplished by decreasing the taper and/or moving the shoulder forward and increasing the shoulder angle.

Two interesting examples of improved cases are the 257 Roberts Improved and the 30-06 Improved. The original 257 Roberts was a modification of the 7X57 Mauser. The improved version of the Roberts not only permitted higher muzzle velocities with the same bullet weights but, with some powders, gave the same velocities as the original Roberts with less powder. The 30-06 Improved, on the other hand, tended to give lower velocities (for the same propellant charge and bullet weight) than the 30-06 and frequently failed to match 30-06 velocities even with increased amounts of powder. Thus, the 30-06 Improved employed what is sometimes referred to as an inefficient case.

There is another condition (related to the inefficient case) which is termed over-bore loading. This condition is most prevalent in small caliber "magnum rounds." In the over-bore situation, the case volume is so large, relative to the bore diameter and length, that there is not time for the propellant to burn completely before the bullet leaves the muzzle. Thus, unburned powder is blown from the muzzle along with the escaping gases. This situation can result when large powder charges are used to obtain high velocities, and, frequently, the powder charge can be reduced without any loss in muzzle velocity. It might appear that a solution is to use a faster burning powder. However, this approach can result in increased peak chamber pressure with rather high probability that the rifle will blow apart.

BULLET DESIGN

It is convenient to consider bullets as comprised of three sections: the nose section (of length λ_n), the (cylindrical bearing) middle section (of length λ_m), and the tail section (of length λ_t).

Sharp-nosed bullets, such as Spitzer and Spire points, have lower drag and, hence, higher ballistic coefficients than round-nosed bullets. Addition of a boattail also tends to reduce drag. It is frequently stated that boattails are important only at lower velocities, say at or below the speed of sound. This does not mean that boattail designs do not decrease drag at the higher velocities. It is simply that the nose drag is so high at the higher velocities that the contribution of a boattail to decreasing overall drag is of little consequence. At lower velocities, the nose drag is greatly decreased and the contribution of a boattail to decreasing overall drag becomes significant. As mentioned in appendix A, all bullets of a given caliber are assumed to have identical nose and tail sections so that differences in weights result from varying the length (λ_m) of the (cylindrical) middle section. It should also be remembered that the tips of sharp-nosed bullets are really slightly blunt, rather than coming to the sharp point dictated by design geometry. The missing sharp tip is sometimes referred to as the meplat (whose length is denoted by δ).

Figure B-1 shows a secant-ogive bullet oriented on the (xy) axes so that the design geometry point is at the origin, the bullet axis lies along the positive y axis, and the plane of the bullet base is at $y = \lambda + \delta$.

λ is the overall length of the bullet,

$$\lambda = \lambda_n + \lambda_m + \lambda_t \quad ,$$

where λ_n, λ_m, and λ_t are the lengths of the nose, middle, and tail sections, respectively, and the nose design length

$$\lambda'_n = \lambda_n + \delta \quad .$$

If all dimensions are expressed in inches, the volumes of the three bullet sections are denoted by V_s {in^3}, (s = n, m, t). Then, for the bullet-axis orientation shown in figure B-1, the formulas are as follows:

$$(x-a)^2 + (y-b)^2 = r^2$$
$$x^2 = a^2 + r^2 - (y-b)^2 - 2a\sqrt{r^2 - (y-b)^2}$$
$$r^2 = a^2 + b^2$$
$$\text{If } \lambda'n < b$$
$$\lambda'n = b - \sqrt{r^2 - (a + d/2)^2}$$
$$\lambda = \lambda n + \lambda m + \lambda t$$
$$\lambda n = \lambda' n - \delta$$

$$\lambda n \, \{d\} = 2.64$$
$$\lambda t \, \{d\} = 0.634$$
$$\delta \, \{d\} = 0.35$$
$$a \, \{d\} = 11.8$$
$$b \, \{d\} = 3.52$$
$$r \, \{d\} = 12.3$$

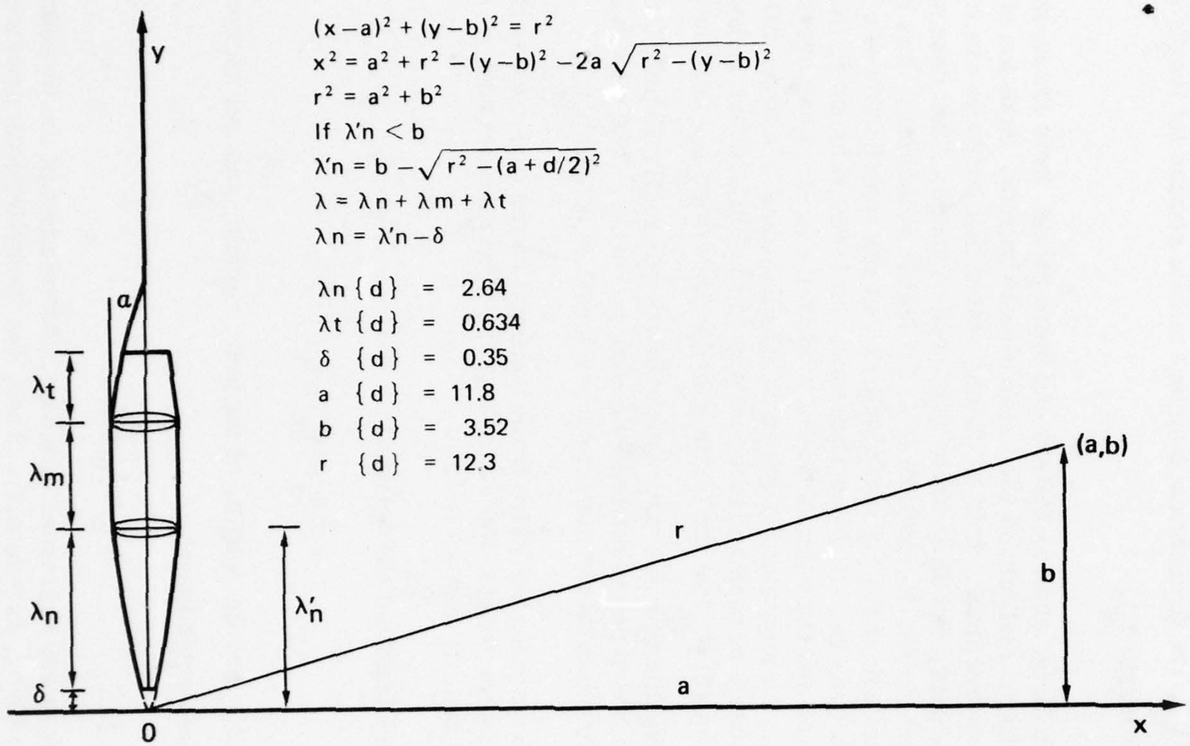

FIG. B-1: SECANT-OGIVE BULLET GEOMETRY

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A METHODOLOGY FOR SELECTING SMALL-ARMS ROUNDS TO MEET MILITARY --ETC(U)
SEP 76 V N BEHRNS N00014-76-C-0001

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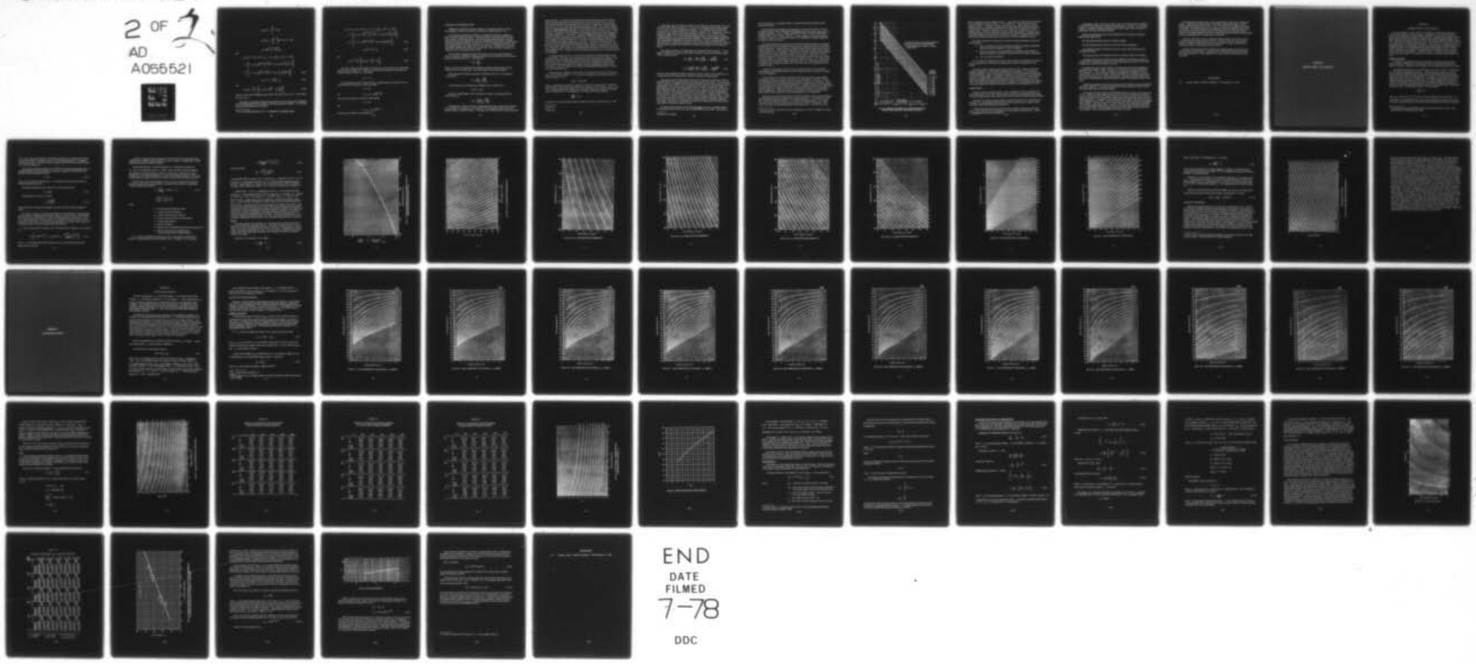

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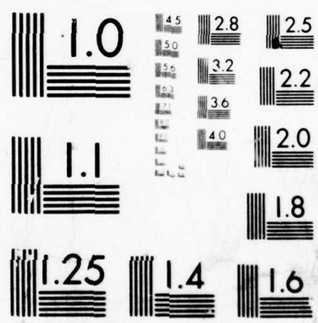

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$$V_n\ \{in^3\} = \pi \int_{\delta}^{\lambda'_n} x^2\, dy \quad ,$$

$$V_n\ \{in^3\} = \pi \int_{\delta}^{\lambda'_n} \Big[(a^2 + r^2) - (y - b)^2$$

$$- 2a \sqrt{r^2 - (y - b)^2} \Big]\, dy \quad ,$$

and

$$V_n\ \{in^3\} = \pi\, (a^2 + r^2)\ (\lambda'_n - \delta) - \frac{\pi}{3} \Big[(\lambda'_n - b)^3 - (\delta - b)^3 \Big]$$

$$- a\pi \left[(\lambda'_n - b) \sqrt{r^2 - (\lambda'_n - b)^2} + r^2 \arcsin \left(\frac{\lambda'_n - b}{r} \right) \right]$$

$$+ a\pi \left[(\delta - b) \sqrt{r^2 - (\delta - b)^2} + r^2 \arcsin \left(\frac{\delta - b}{r} \right) \right] \quad ; \qquad \text{(B-1)}$$

$$V_m\ \{in^3\} = \pi \left(\frac{d}{2} \right)^2 \lambda_m \quad ; \qquad \text{(B-2)}$$

and

$$V_t\ \{in^3\} = \frac{\pi \lambda_t}{3} \left[\left(\lambda_t \tan\alpha - \frac{3d}{4} \right)^2 + \frac{3}{4} \left(\frac{d}{2} \right)^2 \right] \quad , \qquad \text{(B-3)}$$

where (a, b) are the coordinates of the center of the ogive circle, and r is its radius ($r^2 = a^2 + b^2$).[1]

Generally, the formulas relating bullet parameters can be expressed in condensed form if the bullet diameter (d) is taken as the unit of measure of length. The resulting section volume formulas are as follows:

---

[1]Not to be confused with the use of r in appendix D to designate range.

$$V_n \{d^3\} = \pi\, (a^2 + r^2)\, (\lambda'_n - \delta) - \frac{\pi}{3} \left[(\lambda'_n - b)^3 - (\delta - b)^3 \right]$$

$$- a\,\pi \left[(\lambda'_n - b) \sqrt{r^2 - (\lambda'_n - b)^2} + r^2 \arcsin \left(\frac{\lambda'_n - b}{r} \right) \right]$$

$$+ a\,\pi \left[(\delta - b) \sqrt{r^2 - (\delta - b)^2} + r^2 \arcsin \left(\frac{\delta - b}{r} \right) \right] \quad ; \qquad \text{(B-4)}$$

$$V_m \{d^3\} = \frac{\pi}{4}\ \lambda_m \quad ; \qquad \text{(B-5)}$$

and

$$V_t \{d^3\} = \frac{\pi \lambda_t}{3} \left[\left(\lambda_t \tan \alpha - \frac{3}{4} \right)^2 + \frac{3}{16} \right] \quad . \qquad \text{(B-6)}$$

The above volume formulas apply to bullets having a secant-ogive nose and boattail design as shown in figure B-1. They can be applied to secant-ogive nosed bullets not having boattails by setting $V_t = 0$ and taking

$$\lambda_m = \lambda - \lambda_n \quad .$$

Corresponding formulas for bullets having a tangent-ogive nose design can be obtained from the secant-ogive formulas by setting

$$a = r - \frac{d}{2}$$

and

$$b = \lambda'_n = \tfrac{1}{2} \sqrt{d(4r - d)} \quad ,$$

when the unit of measure is the inch, and by setting

$$a = r - \tfrac{1}{2}$$

and

$$b = \lambda'_n = \tfrac{1}{2} \sqrt{4r - 1} \quad ,$$

when the unit of measure is the caliber (d).

ESTIMATES OF REQUIRED TWIST

Rifling twist is stated as the bore length for one complete rotation of a land. For example, a twist of one rotation in 10 inches is usually stated as 1-10.

It is sometimes desirable to estimate the rifling twist required to stabilize a bullet of a given weight and caliber. The determination of the best twist for a rifle bore involves, among other factors, the down-range projectile velocity, which decreases with time at a different rate than does the rotation imparted by the rifling twist. The specific gravity and design of the bullet, as well as atmospheric conditions, also affect bullet stability and, hence, the twist rate required. Furthermore, more than one weight of bullet may be fired from a given rifle. Thus, the whole situation is inclined to end in a compromise rate of twist.

The so-called "Greenhill formula" (reference B-1) has been used to estimate the twist required to stabilize an elongated (jacketed lead-core) bullet having a specific gravity of approximately 10.9. This formula is

$$\tau\{d\} = \frac{150}{\lambda\{d\}} ,$$

where $\tau\{d\}$ is the twist defined as the distance in calibers along the bore for one complete turn of the rifling, and $\lambda\{d\}$ is the length of the bullet in calibers.

If the specific gravity (SG) of the bullet is different from 10.9, the formula for $\tau\{d\}$ becomes

$$\tau\{d\} = \frac{150}{\lambda\{d\}} \sqrt{\frac{SG}{10.9}} .$$

Twist in inches can be obtained by multiplying twist in calibers by d

$$\tau\{in\} = d\ \tau\{d\} .$$

However, if bullet length $\lambda\{in\}$ is expressed in inches, the estimated twist in inches is given by

$$\tau\{in\} = \frac{150\ d^2}{\lambda\{in\}} \sqrt{\frac{SG}{10.9}} .$$

Unfortunately, the above formulas, though appearing to give satisfactory estimates for 30 calibers, do not appear to be satisfactory for estimating the twist required to stabilize bullets of smaller calibers. This failure for smaller calibers may be primarily

velocity related. A situation that developed with the introduction of the 6mm by Winchester and Remington has some implications for the above twist formula. However, some background information relating to this situation is of interest. Factory loading of the 6-mm. Lee Navy cartridge had been discontinued in 1935. Warren Page, the gun editor for Field and Stream, and other varmint hunters found that 6mm was an excellent caliber for long-range varmint shooting. Interest in the 6mm was revived; and, in 1955, Winchester introduced the 243 Winchester and Remington introduced the 244 Remington with bullet diameters d = .243 inches. Apparently Winchester thought of the caliber as suitable for targets ranging from varmints to deer, while Remington conceived of their 6mm as being a varmint cartridge only. The result was that Winchester rifles had a 1-10 twist[1] while Remington rifles had a 1-12 twist.[1] The slower twist (one turn in 12 inches) would not stabilize a 100-grain Spitzer bullet, while the 1-10 twist would. When the 6-mm. gained in popularity as a deer and antelope caliber, the demand increased for heavy 100-grain bullets, which were stabilized by the Winchester's 1-10 twist, but not by the Remington's 1-12. Therefore, the 243 Winchester became much more popular than the 244 Remington.

It is claimed that while the 1-12 twist will not stabilize a 100-grain Spitzer bullet, it will stabilize a 100-grain, round-nose bullet, which is shorter than the Spitzer. The implication is that the difference in lengths of these two nose designs is enough to require a faster twist. It is interesting to note that when Remington reintroduced their new version of the caliber (this time naming it the 6mm Remington) in 1963, the new rifles had a 1-9 twist.[2] However, no evidence indicating that the Winchester 1-10 twist is too slow has been found.

This situation is of interest because if the twist required to stabilize the 105-grain Speer 6-mm. bullet of length λ = 1.11 inches is calculated from the above formula, we obtain (using SG = 10.5),

$$\tau\{in\} = 7.83 \text{ inches} \quad .$$

This is a much faster twist than is required in view of the historical facts. Therefore, the above twist formula must yield a faster than required twist for 6-mm. rounds. However, it appears that these formulas give reasonable results when applied to 30-caliber rounds. Furthermore, if we multiply the 7.83 value by the ratio of calibers,

$$\frac{.308}{.243} = 1.27 \quad ,$$

the result is 9.94, which agrees with the Winchester twist of 1-10 for its 6-mm. rifle.

[1] Model 722.

[2] Model 700.

At the other end of the caliber spectrum, the 30-40 Krag, which was the U.S. Army rifle from 1892 to 1903, employed a 1-10 twist with a 220-grain bullet having a length of 1.35 inches and obtained satisfactory stability. The specific gravity of this bullet is given as 10.9, and the formula for $\tau\{in\}$ gives a twist of 1-10. Furthermore, several current 30-caliber rifles (such as the Winchester Model 70 in Cal. 30-06, and the Weatherby Mark V in Cal. 300 Weatherby magnum) employ 1-10 twists and, according to handloading data and commercial rounds available, successfully stabilize Spitzer and Spire point bullets weighing in the vicinity of 200 grains. Hence, it appears that the above formulas for twist give acceptable answers at the large end of our caliber spectrum (30 caliber) but indicate more twist than is required at the small end of the spectrum (6mm).

The following formulas for estimating twist to stabilize bullets of length λ (which include a caliber ratio) appear to give better estimates of required twist than are given by the Greenhill formula:

$$\tau\{d\} = \frac{150}{\lambda\{d\}} \left(\frac{.308}{d}\right)\sqrt{\frac{SG}{10.9}} \simeq \frac{14\sqrt{SG}}{d\,\lambda\,\{d\}} \quad . \qquad \text{(B-7)}$$

$$\tau\{in\} = \frac{150\, d^2}{\lambda\,\{in\}} \left(\frac{.308}{d}\right) \sqrt{\frac{SG}{10.9}} \simeq \frac{14d\,\sqrt{SG}}{\lambda\{in\}} \quad . \qquad \text{(B-8)}$$

However, their background should be understood and the results of their application, to an estimation of twist requirements, should be considered as approximate.

There appears to be ample evidence that the twist required to stabilize a bullet depends not only on bullet geometry but (among other factors) its velocity. Thus estimates of twist required to stabilize a bullet of given geometry should involve trajectory velocity. Unfortunately, even if the precise rotational velocity required to stabilize a bullet at a given trajectory velocity were determined, there is no way to provide the required variable rotational velocity as the bullet progresses down range at an ever decreasing trajectory velocity. The attainment of such an "exact solution" is further complicated by the fact that the rotational and trajectory velocities decrease at different rates. All that can be done is to impart an initial rotational velocity (through a particular twist) and let it and the muzzle velocity decrease at their respective rates as the bullet progresses down range. Therefore, it is desirable to estimate the required twist by some formula which involves the bullet's muzzle velocity.

An article by John Maynard, in the 1962 Gun Digest, presents a graphical method for estimating required twist which takes muzzle velocity into account. This graph[1] is

[1] Courtesy of Gun Digest.

shown as figure B-2. In using this figure, the bullet diameter and length are both measured in inches.

The experience of Winchester and Remington, with the 243 Winchester, the 244 Remington, and the 6-mm. Remington round stability relative to twist, holds some important implications. First, it appears that both 1-9 and 1-10 twists satisfactorily stabilize heavy, 6-mm. bullets. Secondly, as bullet weight in a particular caliber is increased, situations are encountered where twists acceptable for lighter bullets fail to stabilize the heavier variety.

It is quite evident that no matter how streamlined the bullet geometry, the bullet's (longitudinal) axis must be aligned with the direction of flight to obtain the full benefits of the streamlining. In other words, to minimize drag the bullet should point "head on" along its trajectory. This means that the bullet's axis should coincide with the tangent to the trajectory curve at all points. Any departure from this orientation increases the drag and results in increased time of flight to a given range. Therefore, so far as stability is concerned, the "best twist" rate is that which minimizes the time of flight (to a given range r). Furthermore, if the time of flight to range r is a minimum for a particular round-twist bore length combination, this same combination should result in minimum time of flight for ranges both considerably less than and greater than the particular range r.

If the above reasoning is valid, all that is required in practice is to determine (by carefully controlled tests) the twist which minimizes the time of flight to the ranges of weapon utilization.

No test evidence was found to establish that the gyroscope effect of high rotation velocity (fast twist) was of concern for small-arms projectiles.[1] However, accounts of bullet disintegration attributed to fast twist appear in small-arms literature. In one such experience, the cause of bullet disintegration was finally traced to very thin jackets. The engraving had cut entirely through the jackets. It appears that fast twist, high muzzle velocity, tight bores, high or sharp lands, and thin bullet jackets can all contribute to bullet disintegration. Therefore, the exact contributions of fast twist to gyroscopic effect (with increased drag) and to bullet disintegration are not determined.

The reader may begin to feel that twist should be included in the parameter relations developed for between- and within-caliber round comparisons. To reinforce this feeling, consider the following example. Two rounds are being compared. They satisfy all the parameter relations (developed in appendix A) for valid comparisons. These rounds are fired from appropriate weapons except that round I is fired with a twist

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[1] Excessive twist may contribute to bullet disintegration or to lack of stability according to some authorities.

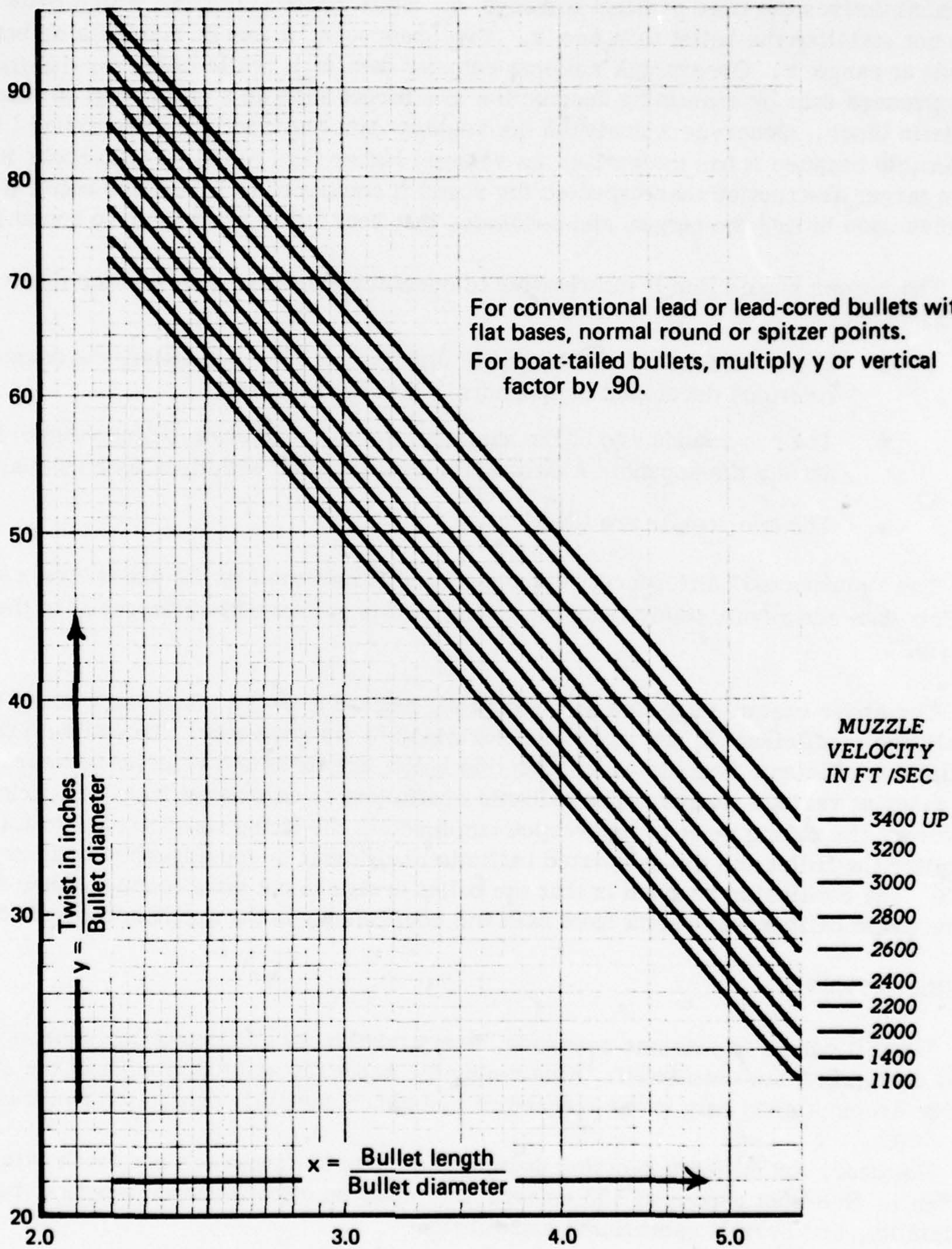

FIG. B-2: ESTIMATE OF TWIST AS A FUNCTION OF BULLET LENGTH AND DIAMETER AND MUZZLE VELOCITY

which minimizes the time of flight to range r, while round II is fired with a twist which does not stabilize the bullet to range r. Two observers, A and B, take data on both rounds at range r. Observer A collects velocity data (v_r), while observer B collects effectiveness data by examining destruction to a target such as a gallon can of water or a gelatin block. Observer A analyzes his velocity data and concludes that round I is preferable because it has maintained its velocity better than round II. Observer B finds more target destruction than expected for round II compared to I (because bullet II has tumbled upon hitting the target) and concludes that round II is preferable to round I.

The reader should find it informative to consider the above example for three separate situations:

- The two rounds (I and II) are of different caliber but satisfy the parameter relations developed in appendix A (are homologous).
- The two rounds are of the same caliber with different bullet weights but satisfy the appendix A parameter relations for within-caliber comparisons.
- The two rounds are identical.

The "unexpected" differences in rounds I and II indicated by the observers' analysis of their data can result solely from the rifling twists rather than differences in the rounds.

The above example exposes another pitfall related to twist. Suppose a bullet has a ballistic coefficient C, say .523 as determined by its geometry. To estimate the ballistic coefficient, rounds loaded with this bullet are fired and velocity measurements are taken at various ranges. The ballistic coefficient is calculated from this firing data. However, the rifling twist of the weapon employed in the firing was too slow to fully stabilize the bullet and the calculated ballistic coefficient is considerably smaller than .523. The conclusion reached is that the bullet design is not good because other designs of the same caliber and weight have ballistic coefficients in the neighborhood of .500.

OBSERVATIONS

When it comes to extreme accuracy, every combination of round components and rifle design is a law unto itself. Fine tuning for accuracy and functioning of the system can be accomplished only in the laboratory and under carefully controlled range conditions.

However, for military systems design, the focus of concern is not on shooting one-half inch, five-shot groups at 100 meters but rather on effectiveness, weights, recoil, reliability, and overall operational capabilities.

The following comments can be found in various places in small-arms literature. Some of these are based on long experience or very carefully controlled tests. Others are aggregated from various implications.

In designing a round, the primer, powder, case, and bullet should be considered in relation to the weapon and its operational requirements. In short, it is a systems problem. Attempts to change the powder type or bullet weight, after the system is operational, are likely to present problems.

Rounds having relatively short overall length tend to work better in automatic weapons than longer rounds.

The case neck should be at least one caliber in length.

The case shoulder angle should be on the order of 20 to 30 degrees.

The propellant charge should fill the case and the base of the bullet should not compress the powder.

The bearing surface of the bullet should have a minimum length of one caliber.

Long-loaded rounds (zero freebore) present difficulties if it should be decided to increase bullet weight after the system becomes operational because the heavier bullet is longer and must be loaded deeper in the case.

It appears that there is some latitude in the rifling twist which can satisfactorily stabilize a bullet of given design. However, too much twist may make the bullet act like a gyroscope -- if so, it tends to maintain its original axis orientation, which increases drag. Too little twist tends to permit the bullet axis to oscillate about the tangent to the trajectory curve, which increases drag. Increasing the bullet weight after the system is operational may generate stability problems if the weapon employs a slow twist.

Boattail bullet designs tend to decrease drag, but the contribution becomes important only at low velocities. It is sometimes stated that boattail designs are less perturbed (by muzzle blast) than are flat-base designs.

The secant-nose bullet design was standardized for the 7.62-mm. NATO round, and the U.S. employs a secant design in the M14 rifle. Some test reports tend to show superiority of the secant over the tangent design, while others cast doubt on the existence of this superiority. For example, three 180-grain 30-06 bullets were tested by the H.P. White Laboratory for the NRA. One of these was a secant-ogive, while two were tangent-ogive designs. The mean velocity loss (to 300 yards) for the secant-ogive was greater than for either tangent-ogive bullet. However, the secant-ogive bullet had a flat base and rounded-nose tip, while both tangent-ogive bullets had rather sharp points and one had a boattail. Thus, the implications of the test are not clear.

The conditions for testing secant versus tangent bullet designs are not sufficiently defined. Certainly the bullets should be of the same calibers and weights, and the bullet material used should be the same. Also, the bullet tails and rifling twist should be identical. The secant design has two parameters, while the tangent has one and the exact relations among the parameters for valid comparisons have not been defined. However, both ogive designs are employed in bullets having good ballistics characteristics and valid comparisons are of concern only to the purist.

The term "over-bore" is used to describe a situation where not all of the powder is burned before the bullet leaves the muzzle. In this situation, the case-bore combination is inefficient because the ratio of muzzle energy to energy in the powder is lower than normal. Such designs should be avoided.

Usually, the best performance is obtained with lighter bullets in small cases and heavier bullets in larger cases. "Magnum rounds" employing relatively large cases have been successful, but tests indicate that for small calibers most such rounds are over-bore.

REFERENCES

B-1. Hatcher, Julian, "Hatcher's Notebook," The Stockpole Co., 1966

APPENDIX C

IMPULSE, RECOIL, AND ACCURACY

APPENDIX C

IMPULSE, RECOIL, AND ACCURACY

The terms impulse, recoil, and kick are frequently loosely used to describe the action on the shooter from firing a weapon. Unfortunately, each of the three terms leaves much to desired as a measure of this reaction. Impulse is more properly termed recoil impulse or recoil momentum; its effect on the rifleman depends on weight of the rifle. Once the rifle weight is determined, the recoil (more correctly the free recoil) can be calculated from the impulse and rifle weight. The term kick is considered to involve the characteristics of the cartridge, the rifle, and the rifleman, including the weight and design of the rifle, and the firing position and the physical structure of the rifleman. In these respects, it is a measure of the punishment the rifleman receives. Unfortunately, kick cannot be quantified -- at least at present. Therefore, for purposes of this research contribution, the concept of kick will not be discussed further (reference C-1).

Impulse And Recoil

The units for impulse are the units of momentum; we shall use pound-seconds. Since momentum is conserved in the firing of a rifle, the momentum of the rifle is equal and opposite in direction to the momentum of the material exiting from the muzzle.

The material exiting from the muzzle consists of not only the bullet, but also gases and, sometimes, partially burned powder. The partially burned powder is present when the powder burning rate[1] is such that some of the powder in the round does not have time to burn before the bullet exits from the muzzle. This condition is sometimes referred to as an "over-bore" loading and is most frequently encountered with small-caliber rounds having large powder charges. To further complicate the situation, the velocities of the bullet, the powder gas, and any unburned powder are each different. In fact, some of the powder gas leaves the muzzle at a considerably higher velocity than the bullet. Thus, we could define impulse J by the relation

$$J = \sum_i m_i v_i \quad ,$$

where m_i is the mass of the i^{th} type of material exiting from the muzzle and v_i is its velocity. The trouble with this is that, while it looks good in theory, there are factors such as the temperature of the powder that affect the various velocities (reference

[1] The burning rate varies with pressure and powder characteristics, such as web in tubular powder or coatings in ball powder.

C-1). Also, although the impulse is sometimes referred to as a function of the round, the muzzle velocity of the bullet depends on the bore length (reference C-1) together with the case volume. Therefore, while the weight of the rifle does not affect the value of J, the bore length does.

This rather complicated situation has resulted in two separate approaches to estimating the impulse. Since momentum is conserved, impulse can be estimated from laboratory data employing the formula

$$J = MV \quad , \qquad \text{(C-1)}$$

where M is the mass of the rifle and V is its velocity when fired in some form of cradle permitting free recoil.

The kinetic energy of the free recoil (R) is given by the formula

$$R = \tfrac{1}{2} MV^2 \quad . \qquad \text{(C-2)}$$

From formulas C-1 and C-2, we have

$$J = \sqrt{\frac{2\,W\,R}{g}} \quad , \qquad \text{(C-3)}$$

which can be used to calculate the impulse when the free recoil and rifle weight are known.

The second approach to estimating the impulse is to theoretically or experimentally develop a formula relating impulse to bullet weight and muzzle velocity. Such formulas can be derived from rifle velocity-of-recoil (V) formulas given in references C-1 and C-2. These formulas are not appropriate for our purposes, however, because they were developed for older rifles, such as the Krag, Springfield, and Garand, and do not yield accurate estimates for modern rifles firing modern rounds.

A more recent formula for impulse (J), developed by BRL (reference C-3), is given by:

$$J = w\left[-\,2.48(10^{-10})\,v_o^{\,2} + 4.45(10^{-6})\,v_o + \frac{.0140\,v_o - 10.12}{6823 - v_o}\right], \qquad \text{(C-4)}$$

where w is the weight of the bullet in grains, and v_o is the muzzle velocity of the bullet in feet per second.

Formula (C-4) gives accurate estimates for the M16 and M14 rounds and appears satisfactory for calibers over the range from 5.56 to 7.62mm. Consequently, we shall employ this formula to calculate impulses.

It should be noted that J becomes infinite at $v_o = 6,823$ feet per second and $J < o$ for $v_o > 6,823$ feet per second. However, this is of little concern for present-day small-arms muzzle velocities, which are usually under 4,000 feet per second. Furthermore, for military applications, at least with conventional rounds, barrel wear probably becomes excessive for muzzle velocities beyond about 3,500 feet per second.

The BRL formula for recoil impulse is based on theory combined with measured recoil for various small-arms projectiles to establish semi-empirical relationships. The basic equations used follow:

$$J = \frac{w\, v_o}{7,000g} + 0.023\,(1 - \epsilon)\, c \qquad \text{(C-5)}$$

$$\epsilon = \left(\frac{w\, v_o^2}{2g}\right)\left(\frac{\gamma - 1}{f\, c}\right),$$

where

J = recoil momentum (pound-seconds),

w = weight of projectile (grains),

v_o = muzzle velocity (feet per second),

g = gravity constant (feet per second squared),

c = charge weight (grains),

ϵ = ballistic efficiency,

f = specific force of propellant (foot-pounds per pound), and

γ = ratio of specific heats of propellant gas (constant pressure over constant volume)

If f is taken as 360,000 foot-pounds per pound, representing an idealized propellant as an approximation for many situations, and γ is taken as 1.25, we obtain:

$$\epsilon = \frac{2.48 \times 10^{-10}}{0.023} \left(\frac{w\, v_o^{3}}{c} \right) . \qquad \text{(C-6)}$$

Also, the relation

$$\frac{w}{c} = \frac{11,189 - 1.64\, v_o}{v_o - 758} \qquad \text{(C-7)}$$

was developed by BRL to express $\frac{w}{c}$ as a function of v_o. Inserting C-6 and C-7 in C-5, we obtain C-4, expressing the impulse (J) as a function of bullet weight and muzzle velocity. Using formula C-4, values of J/w were calculated for muzzle velocities in the range 2,500-3,500 feet per second. J/w is shown as a function of v_o in figure C-1.

Combinations of w and v_o yielding fixed values of J can be determined using formula C-4. The resulting curves are shown in figures C-2.1 through C-2.5. Figures C-2.6 and C-2.7 show J as a function of w for given values of v_o down to 2,000 feet per second. These curves serve two purposes: to estimate the impulse, given w and v_o, and to determine combinations of bullet weight and muzzle velocity that yield a given impulse. It should be kept in mind that impulse is a function of the round, in the sense that it depends on the mass and velocity of the material exiting from the muzzle. Furthermore, while the velocity of this exiting material does depend on the length of the rifle barrel, impulse is independent of the rifle weight and, therefore, is not a direct measure of the recoil experienced by the rifleman. However, because impulse is a measure frequently associated with the round only, it has gained acceptance as a round descriptor, independent of rifle weight, and is frequently encountered in the literature.

Recoil

As discussed above, the momentum of a weapon and of the material exiting from its muzzle are equal but oppositely directed. The kinetic energies, however, are not equal; if they were, the weapon would be lethal at both ends. In speaking of a weapon's recoil, it is usual to calculate free recoil. This is the kinetic energy developed by the weapon when fired without other mass (such as the shooter) to impede the rearward motion of the weapon.

If formula C-3 is solved for R, we obtain

$$R = \frac{g}{2W} J^2 , \qquad \text{(C-8)}$$

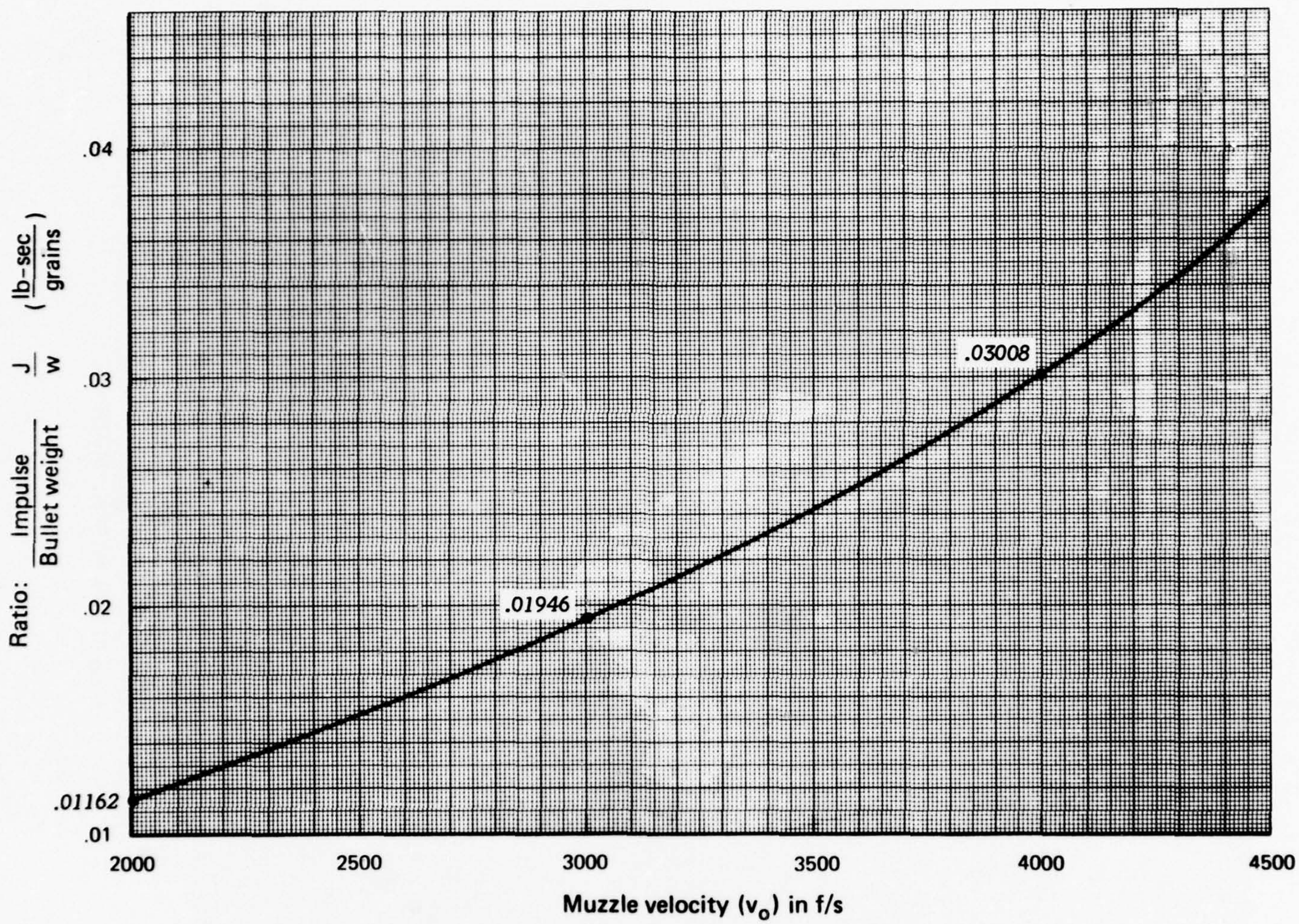

FIG. C-1: RATIO OF IMPULSE TO BULLET WEIGHT (J/w) AS A FUNCTION OF MUZZLE VELOCITY (v_o)

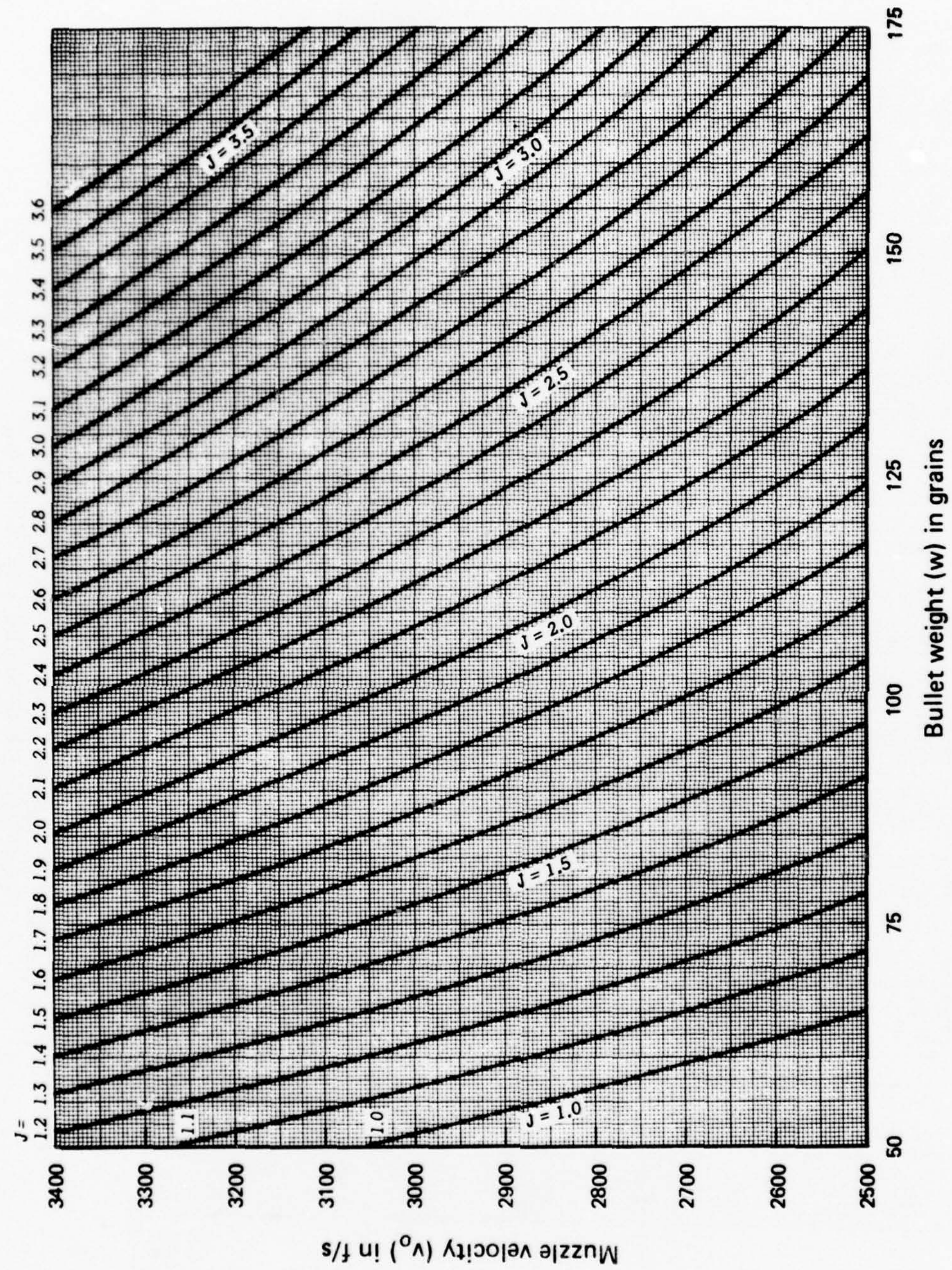

FIG. C-2.1: $w\,v_o$ TRADE OFFS FOR CONSTANT J

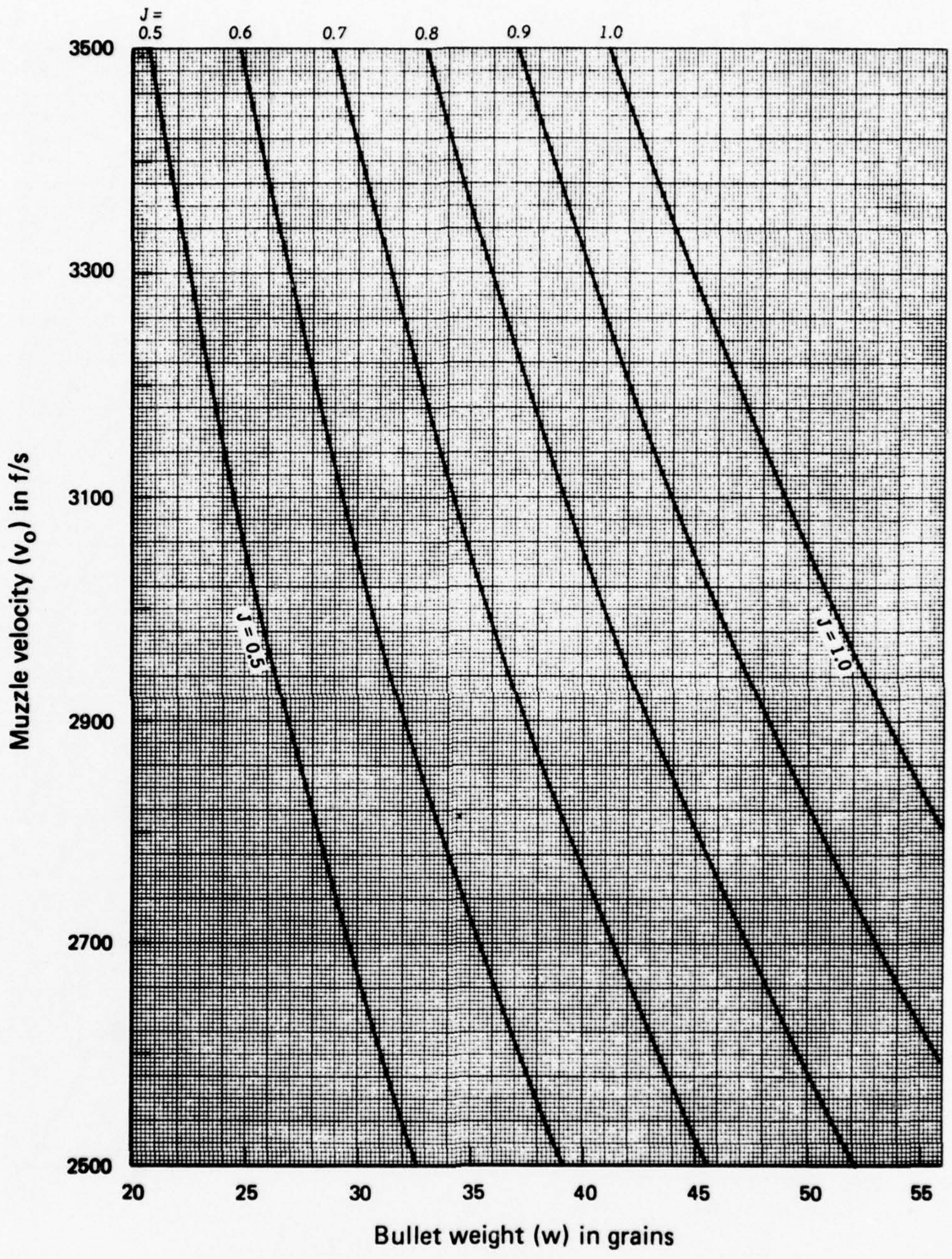

FIG. C-2.2: w v_o TRADE OFFS FOR CONSTANT J

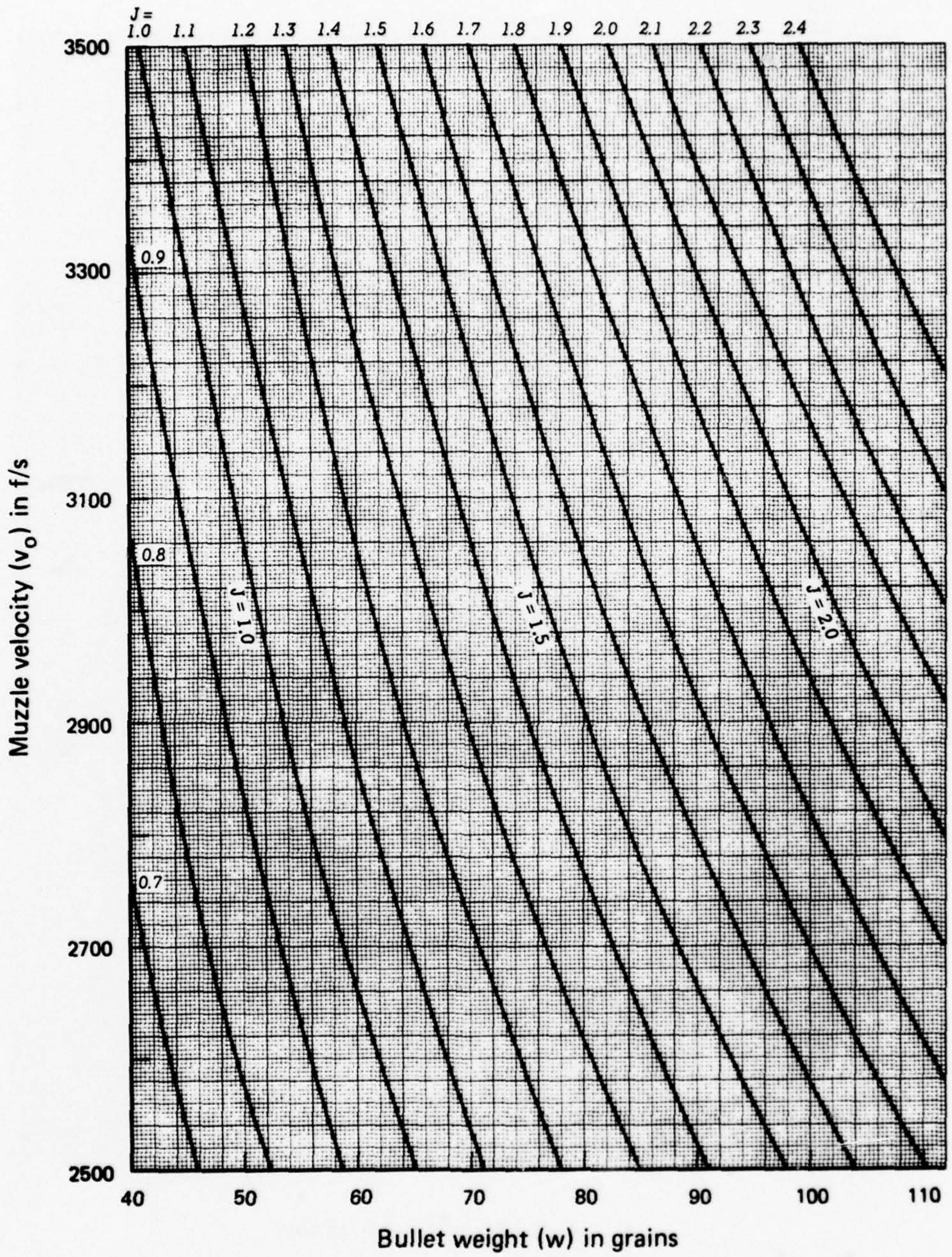

FIG. C-2.3: w v_o TRADE OFFS FOR CONSTANT J

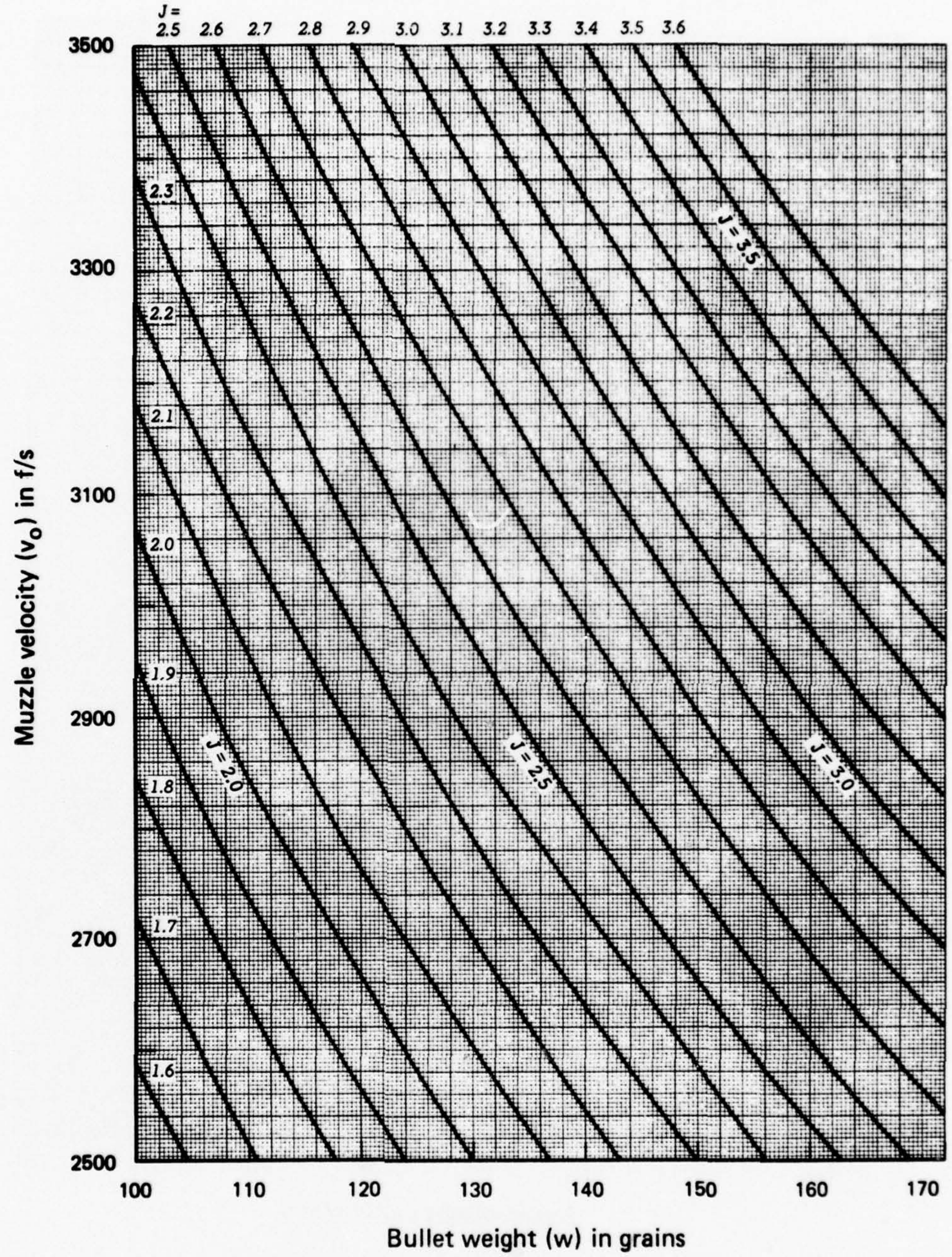

FIG. C-2.4: w v_o TRADE OFFS FOR CONSTANT J

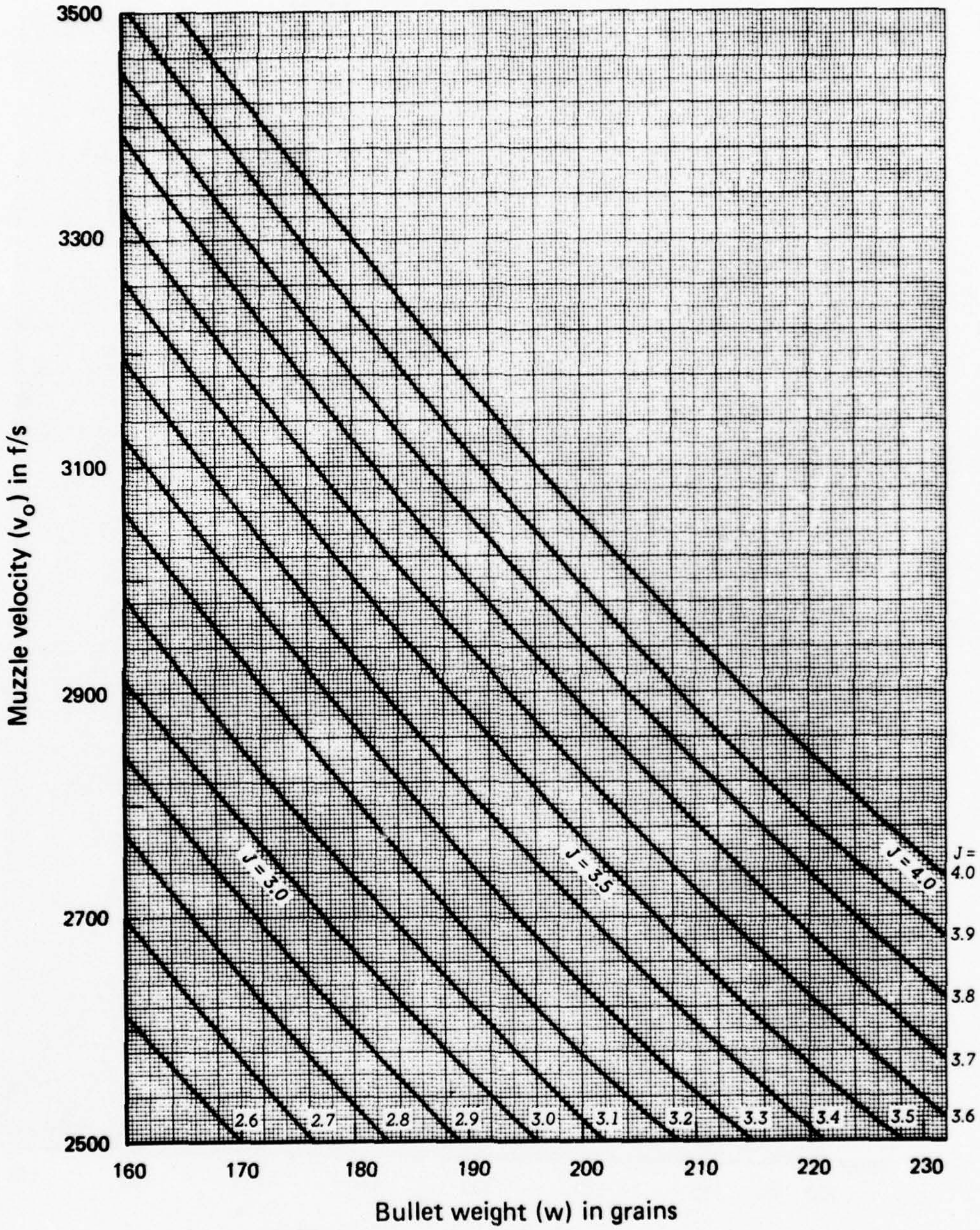

FIG. C-2.5: w v_o TRADE OFFS FOR CONSTANT J

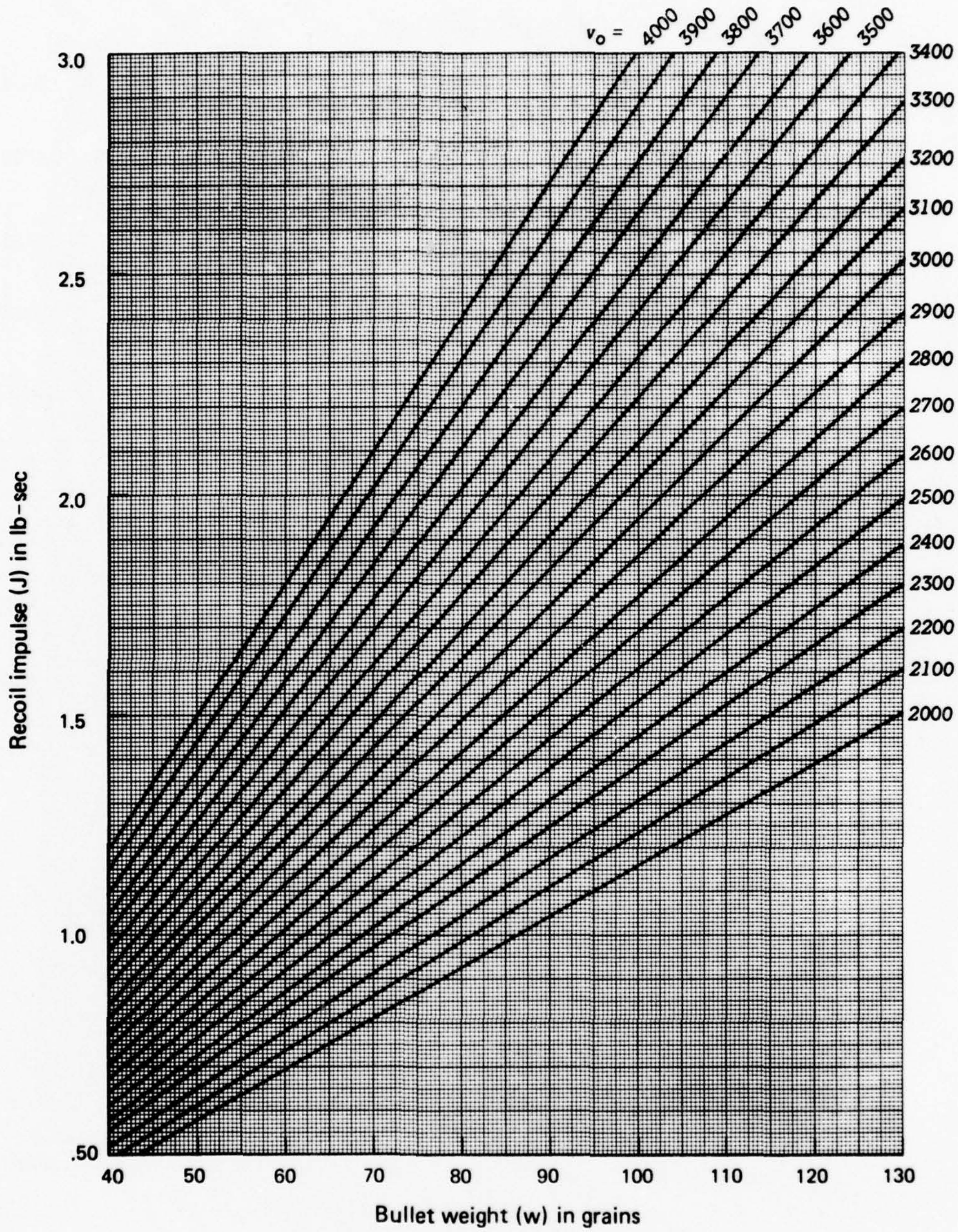

FIG. C-2.6: J AS A FUNCTION OF w FOR GIVEN v_o

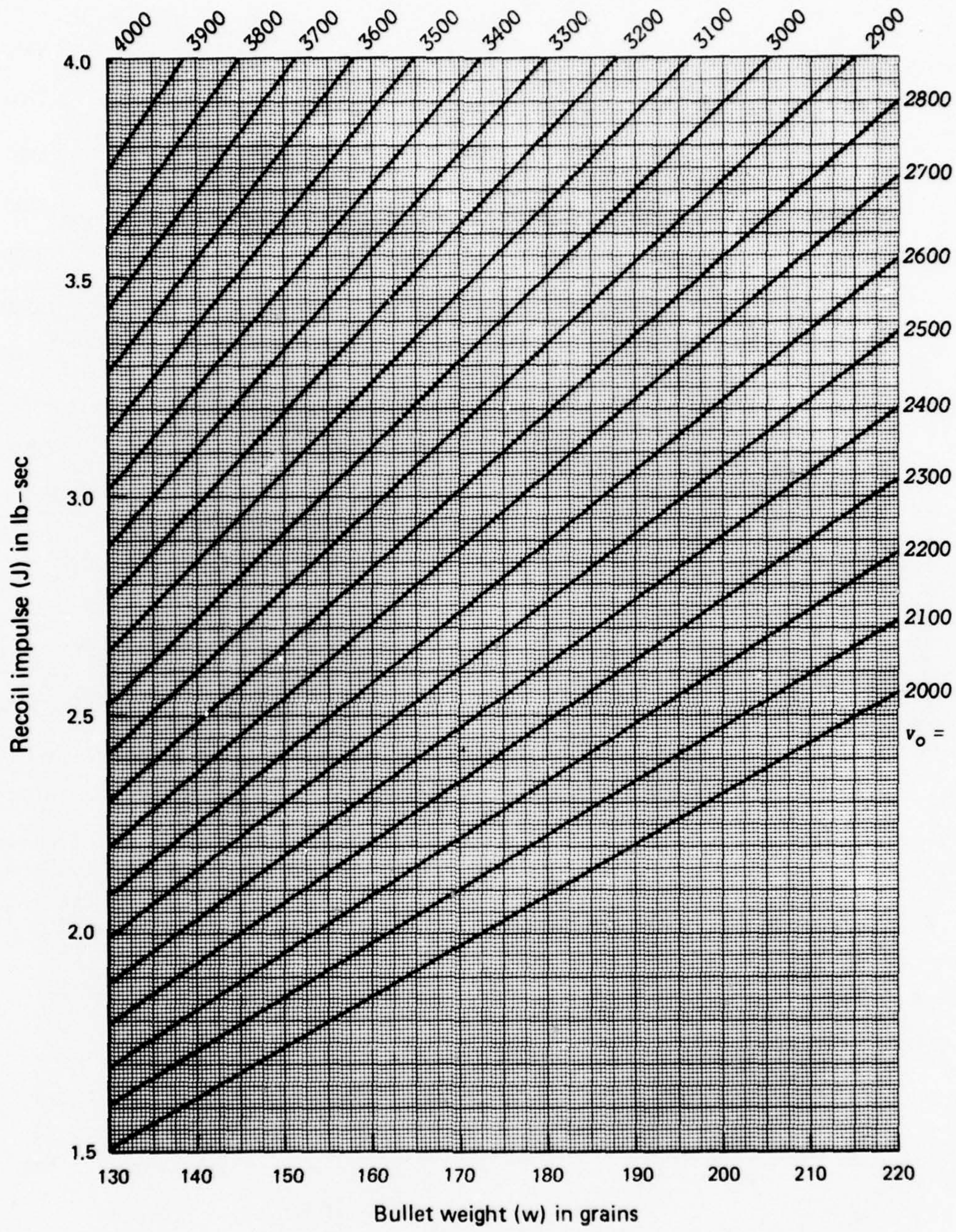

FIG. C-2.7: J AS A FUNCTION OF w FOR GIVEN v_o

and, if 32.16 ft/sec.^{2} is substituted for g we obtain,

$$R = \frac{16.08}{W} J^2 . \qquad \text{(C-9)}$$

Thus, the free recoil (R) of a weapon weighing W pounds, is one half the acceleration of gravity divided by the weight (W), all multiplied by the square of the recoil impulse (J) in pound-seconds.

If values of R from formula C-9 are plotted as a function of W for given values of J, the resulting set of curves can be used to estimate free recoil (R) given the weapon weights (W) and recoil impulses (J). Such curves can also be used to trade off R and W for fixed recoil impulse (J). The resulting curves are shown in figure C-3.

If the same round is fired in two rifles of weights W_1 and W_2, the free recoils (R_1 and R_2) are inversely related to the weights, assuming that J is fixed;

$$R_1W_1 = R_2W_2 \text{ , for fixed } J \quad . \qquad \text{(C-10)}$$

ACCURACY AND RECOIL

There are two distinct aspects of accuracy that are affected by recoil. The first is the climb of a weapon when fired in the automatic or burst-mode. The analysis of climb is beyond the scope of this research contribution, however, and will be dealt with in a subsequent report. The second aspect is primarily associated with the semiautomatic mode of fire (reference C-1, Chapters XI, XII) and results from flinching and perhaps also jerking the trigger caused by the shooter's anticipation of kick from the weapon.[1] Experienced shooters can develop this tendency even when accustomed to a weapon recoil. One of the criticisms of the M-1 (Garand) and M-14 was that these weapons delivered more recoil than was comfortable for some riflemen. Of course, the rifle weights could have been increased, but the weights were already considered by some shooters to be greater than desirable. One solution to this situation is to go to a less

[1] A more common reason for jerking the trigger is to get the round off while the sight picture is good -- but this situation is not recoil dependent.

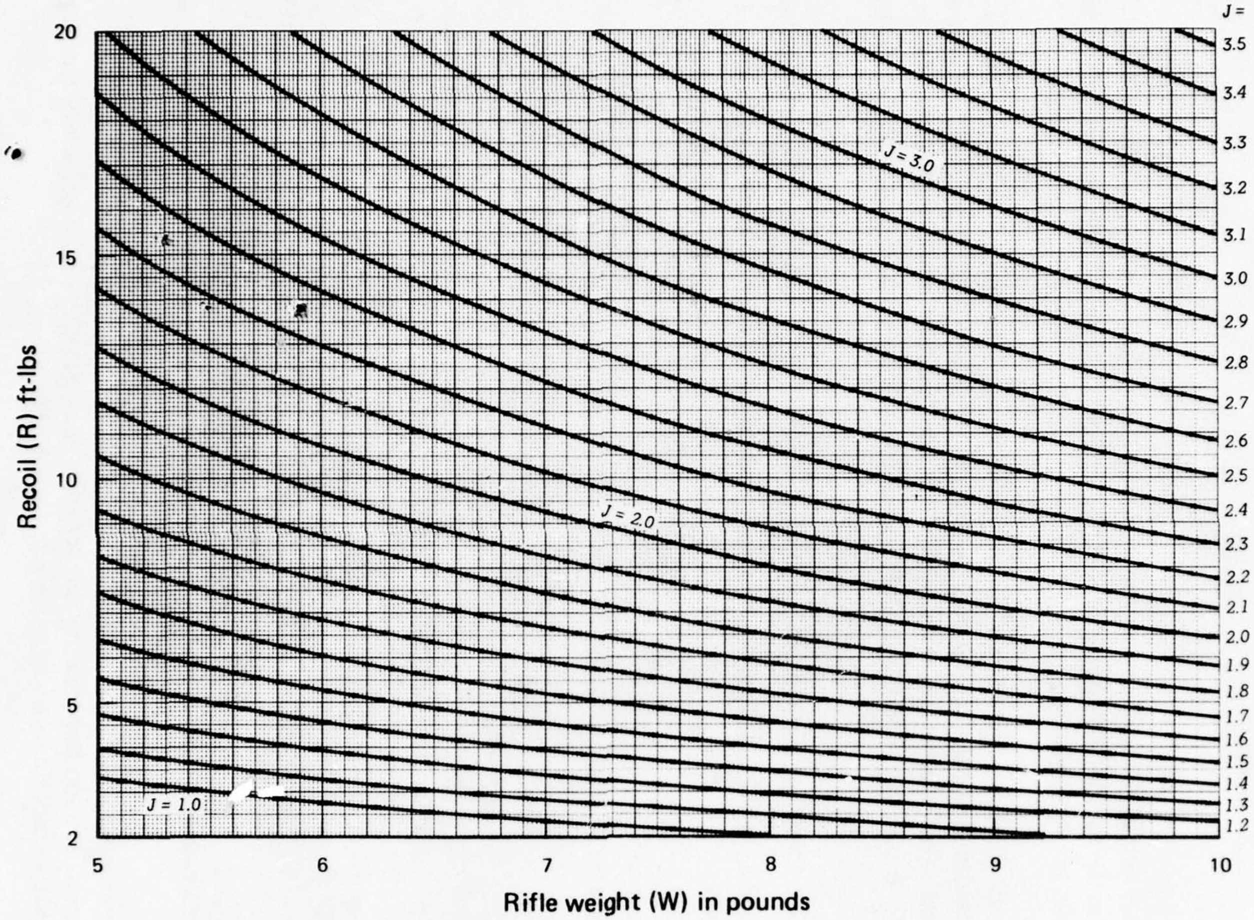

FIG. C-3: R AS A FUNCTION OF W FOR GIVEN J

potent round which was done with the 5.56mm for the M-16 rifle. Since the tendency to flinch or jerk the trigger is so highly dependent on the shooter, extensive field experimentation is required to determine the maximum recoil that the average rifleman can tolerate in various firing postures. Further refinement to estimate the "optimum recoil" when down-range kinetic energy requirements are introduced complicates the problem still more. Such determinations are beyond the scope of this paper. However, there are some guidelines, based on experience, which are worth mentioning. Hatcher (reference C-1, pages 280, 290) states that the maximum energy of recoil for a military rifle should not exceed 15 foot-pounds and estimates the recoil of the Springfield M-1903 as approximately 15 foot-pounds. While 15 foot-pounds of recoil energy may very well have been an acceptable upper bound (to Hatcher) at the time of writing, it most certainly is not generally acceptable today. The M-14 recoil, based on unloaded rifle weight, is on the order of 11 foot-pounds and is frequently referred to as being too severe. The M-16 (unloaded weight) recoil is well under 4 foot-pounds, and it is highly unlikely that a recoil energy in excess of 10-12 foot-pounds will be acceptable in future rifles. Of course, increasing the rifle weight reduces the recoil, but it also is unlikely that a proposed future rifle weight much in excess of 8 or 9 pounds will meet with acceptance. By formula C-3, an 8-pound rifle with 11 foot-pounds of recoil implies a recoil impulse of 2.3 pound-seconds. The implication is that it is desirable to have the recoil impulse of future rifle rounds below approximately 2^+ pound-seconds. In fact, some "authorities" have expressed the belief that the rifle round impulse should be below 1 pound-second. The basis of such statements is hard to understand. Most riflemen consider the 243 Winchester to be a rather mild cartridge, and a 243 round, with a 100-grain bullet and a muzzle velocity equal to 3,070 feet per second, has an impulse of approximately 2 pound-seconds. When fired in a rifle weighing 7 pounds, this round develops a free recoil of approximately 9.3 foot-pounds.

APPENDIX D

DOWN-RANGE VELOCITY

APPENDIX D

DOWN-RANGE VELOCITY

The down-range velocity v_r of a bullet at range r is a function of the muzzle velocity v_o, the ballistic coefficient C, and the range r. After a bullet leaves the muzzle, the force of gravity pulls it towards the ground, and air resistance retards its flight. The resulting bullet path is called the trajectory. The initial part of the trajectory appears rather flat, but soon the curvature becomes greater as the bullet approaches its maximum height H, after which the bullet drops rather rapidly per unit of distance traveled.

The retardation of the bullet by air resistance is of considerable magnitude. For example, the .30-caliber 1906 (30-06) bullet with v_o = 2, 700 feet per second loses 219 feet per second during the 0.12 seconds required to travel its initial 100 yards. This deceleration is at the rate of 1, 825 feet per second per second. Thus, the average magnitude of the deceleration caused by air resistance in the first 100 yards is almost 57 times as great as the acceleration of gravity. This retardation decreases with a decrease in the bullet velocity but not in a simple manner. Thus, ballistic tables (reference D-1, Chapter XXIII) or computers employing numerical methods to solve simultaneous differential equations are employed to carry out exterior ballistics calculations. As stated above, this paper employs Ingalls Tables for trajectory calculations.

The first consideration is to calculate the bullet's velocity v_r at range r, given the muzzle velocity v_o and the ballistic coefficient C.

The formula for use with Ingalls Tables is

$$S(v) = S(v_o) + \frac{r}{C} \quad , \qquad \text{(D-1)}$$

where S(v) is the tabulated value of the velocity function at range r (tabulated as S(u)). $S(v_o)$ is the tabulated value of the velocity function for muzzle velocity v_o, r is range measured in feet, and C is the ballistic coefficient. Once S(v) is calculated from formula D-1, the corresponding velocity v_r is read from the table. Since S(v) does not depend explicitly on bullet weight, the down-range velocity can be calculated as a function of v_o, r, and C. Then, for a given v_o, v can be plotted as a function of C with r parameterized.

The resulting set of down-range velocity graphs, v_o = 2,500(100)3,500 fps, r = 0(100)1,500 meters, follows as figures D-1.1 through D-1.11, with velocity given in feet per second and range given in meters.

TRAJECTORY CHARACTERISTICS

There are several trajectory characteristics which are of interest to the rifleman. The first two to be discussed (maximum height and drop) are frequently employed as measures of trajectory flatness. The term "flat trajectory" is often applied to describe the flight characteristics for bullets whose trajectory does not "rise high" above the horizontal line of sight in hitting a target at a "reasonable range."

Height Of Trajectory

The (maximum) height of trajectory H_{300}, in inches, that a bullet rises above a horizontal line of sight before returning to the original muzzle height, at a range of 300 meters, is of interest because it is desirable to be able to zero a rifle at a single range and have the capability of hitting appropriate targets at all reasonable ranges without changing the sight setting. The formulas associated with calculation of H , using Ingalls Tables, follow.

If T_r is the time of flight for a bullet to some range (r) of concern, then

$$T_r = C\,[\,T(v_r) - T(v_o)\,] \quad , \tag{D-2}$$

where T_r is in seconds and C is the ballistic coefficient; $T(v_r)$ and $T(v_o)$ are time functions found under the Ingalls Table column headed T(u); v_o is the muzzle velocity; and, v_r is the velocity at range r .

Once the time of flight (T_r) is calculated from D-2, the maximum height (H_r) for a bullet aimed to hit a muzzle-high target at range r is given by

$$H_r = 48T_r^{\,2} \quad ,^{1} \tag{D-3}$$

where H_r is the maximum trajectory height in inches.[2]

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[1] T^2_{300} can be found from figure D-3.

[2] Usually referred to as mid-range trajectory though the maximum height occurs beyond the mid-range.

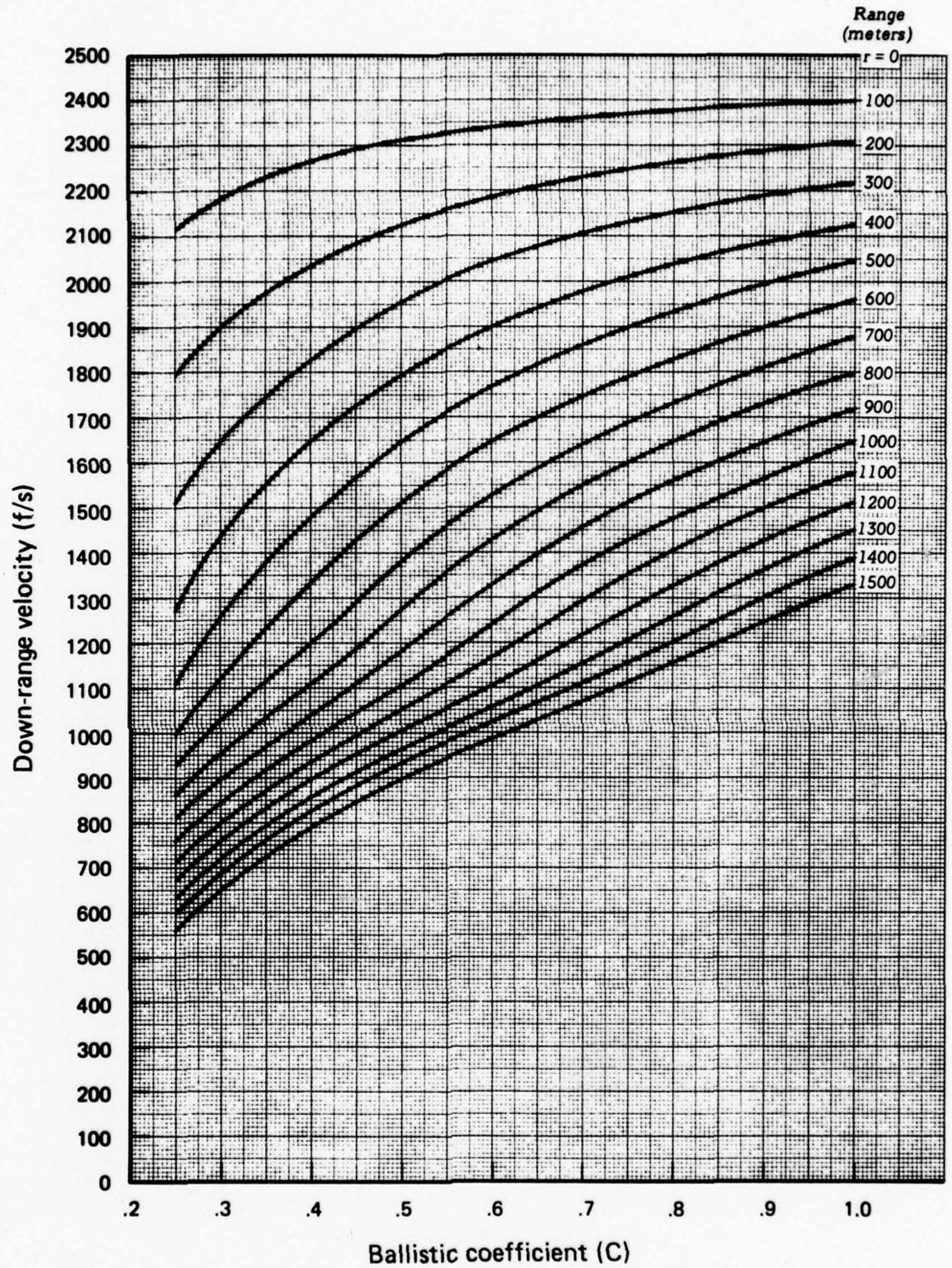

FIG. D-1.1: v AS A FUNCTION OF C FOR GIVEN r, v_o = 2500 f/s

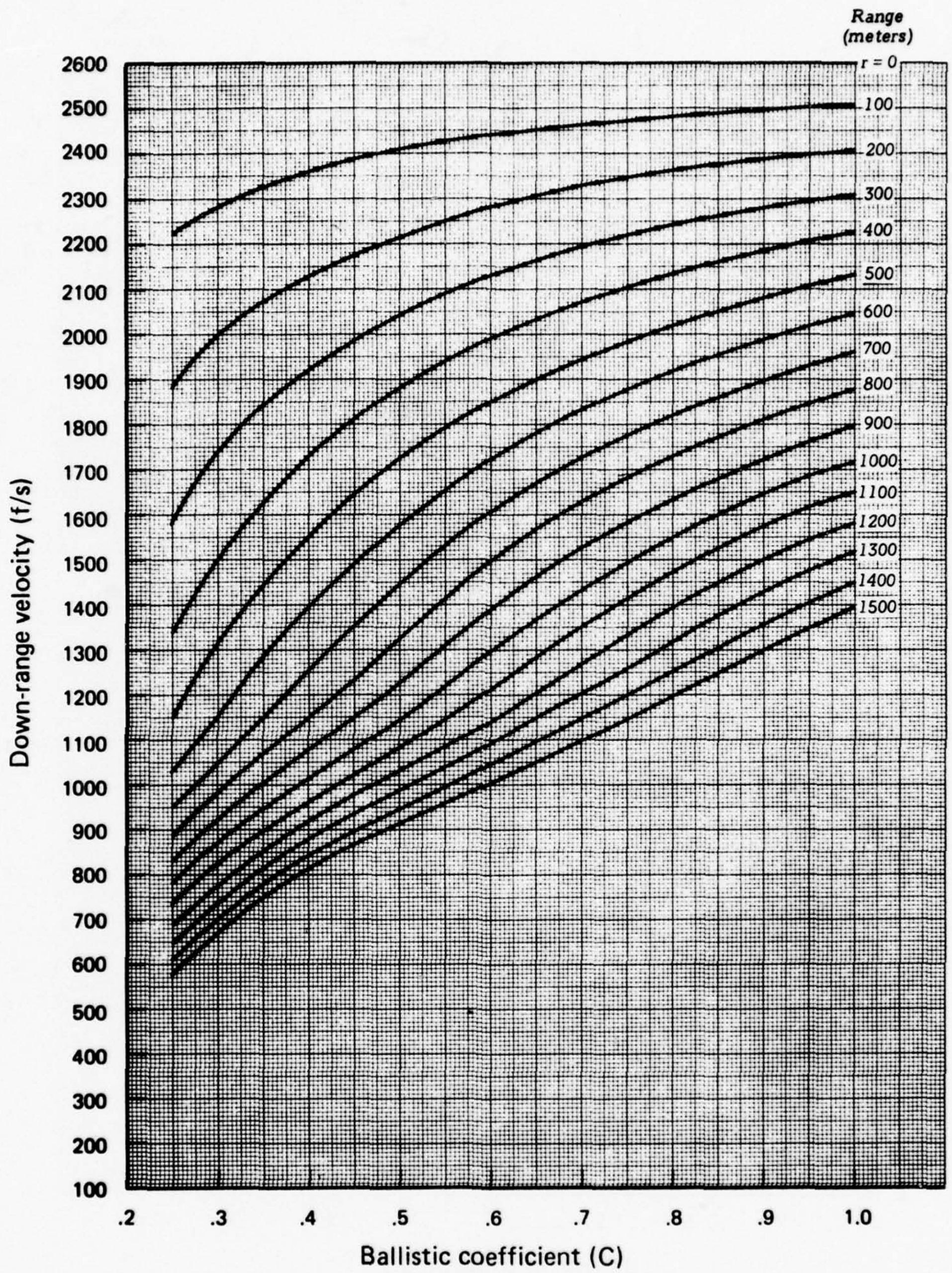

FIG. D-1.2: v AS A FUNCTION OF C FOR GIVEN r, v_o = 2600 f/s

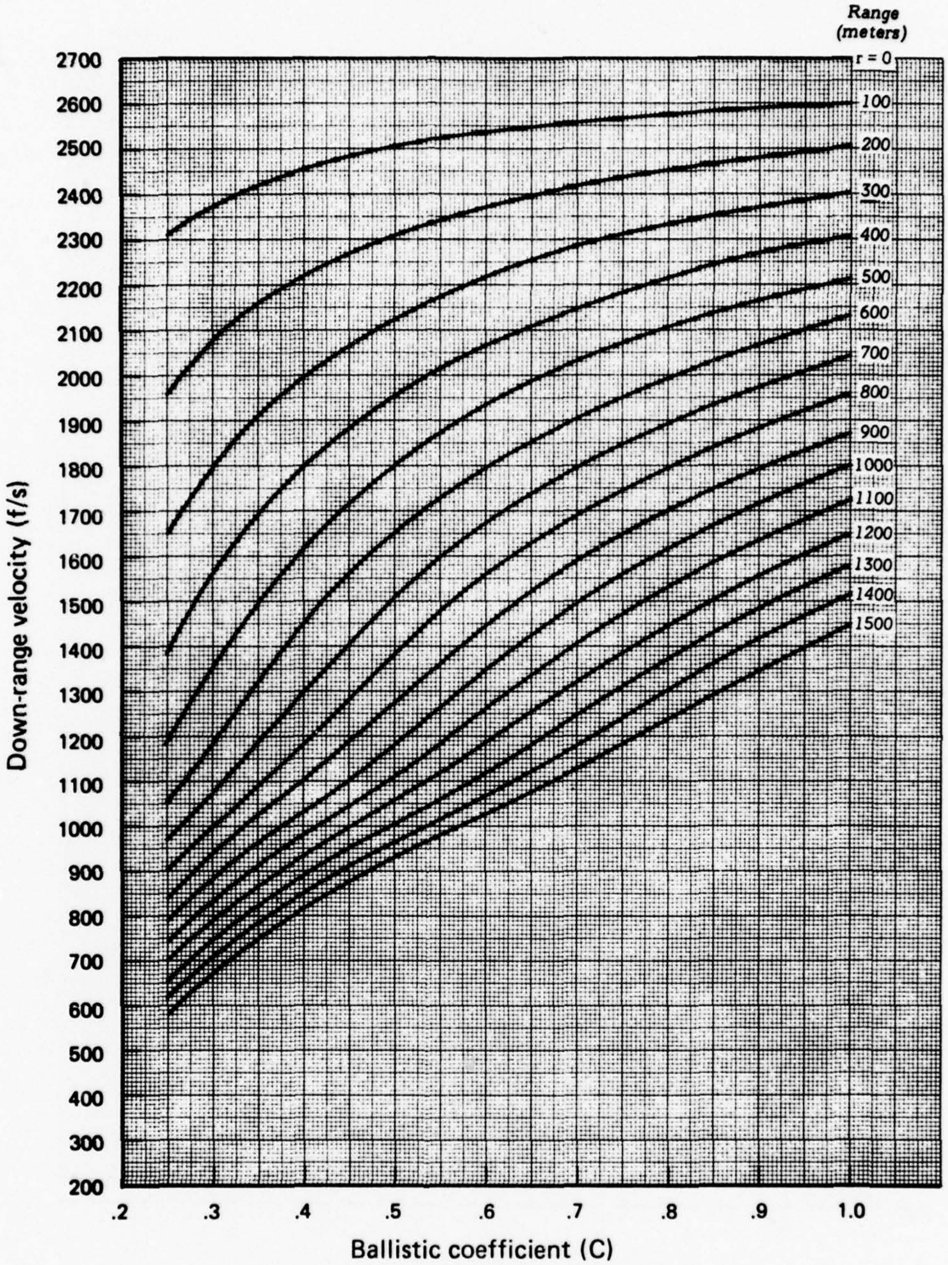

FIG. D-1.3: v AS A FUNCTION OF C FOR GIVEN r, v_o = 2700 f/s

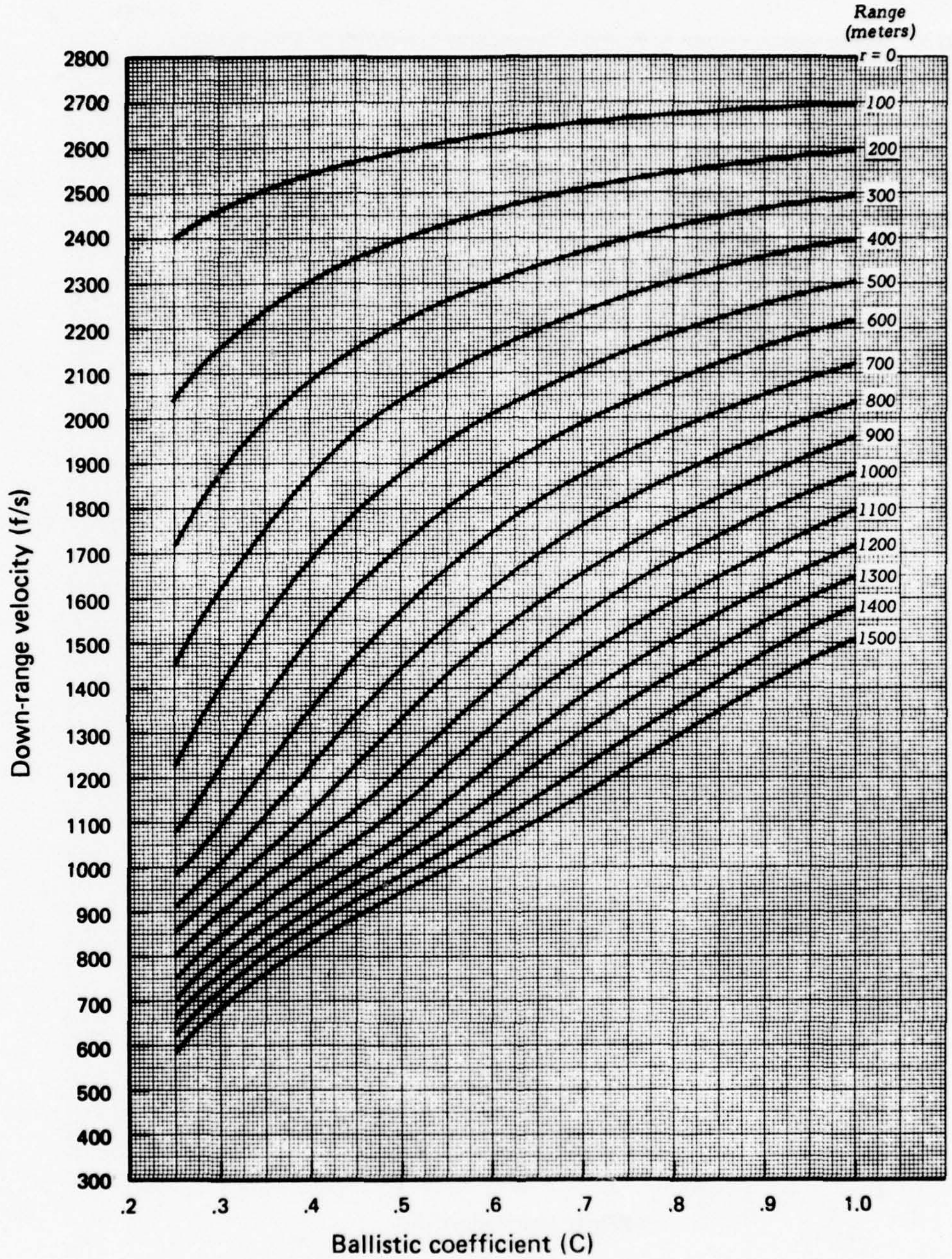

FIG. D-1.4: v AS A FUNCTION OF C FOR GIVEN r, v_o = 2800 f/s

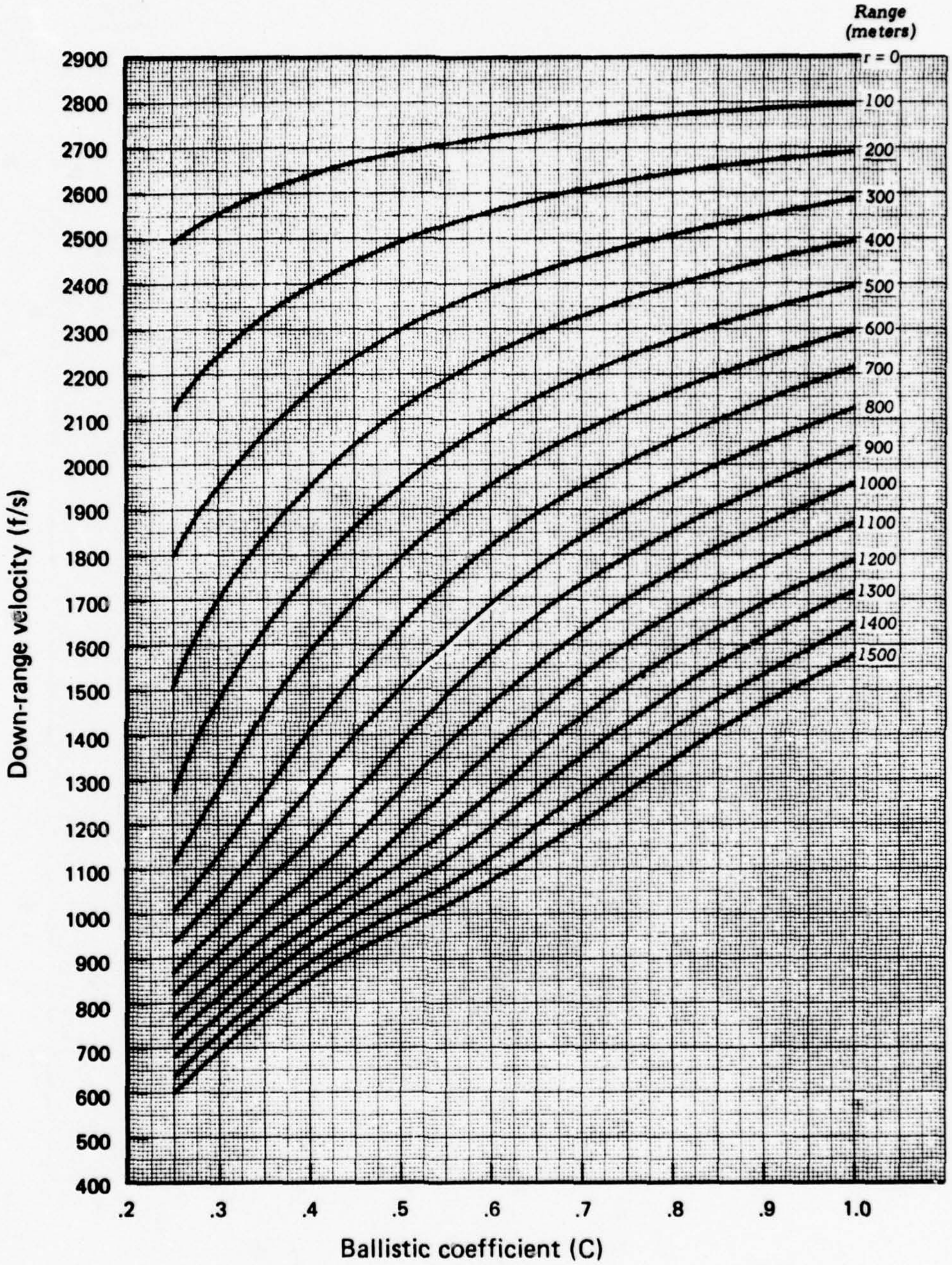

FIG. D-1.5: v AS A FUNCTION OF C FOR GIVEN r, v_o = 2900 f/s

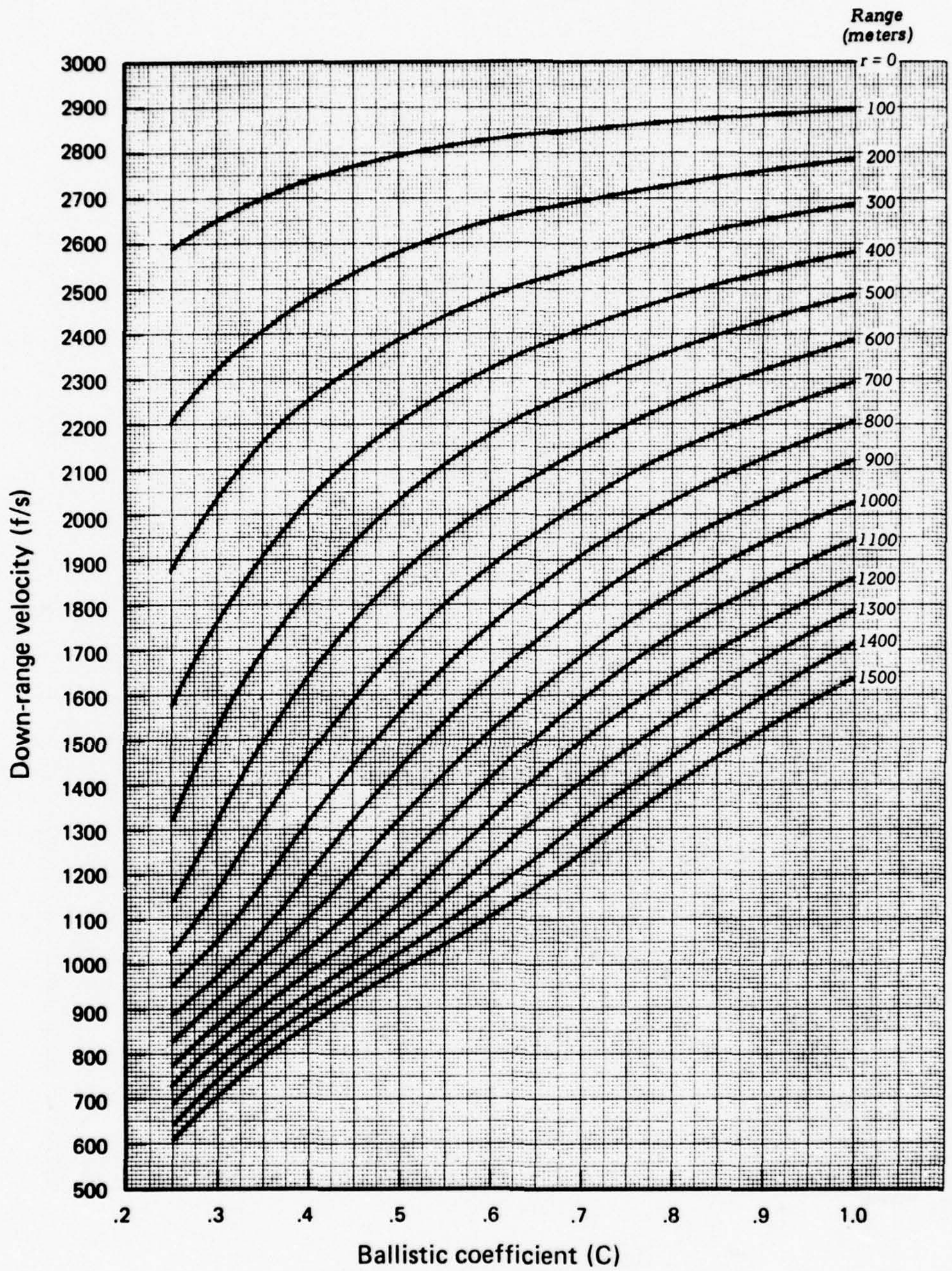

FIG. D-1.6: v AS A FUNCTION OF C FOR GIVEN r, v_o = 3000 f/s

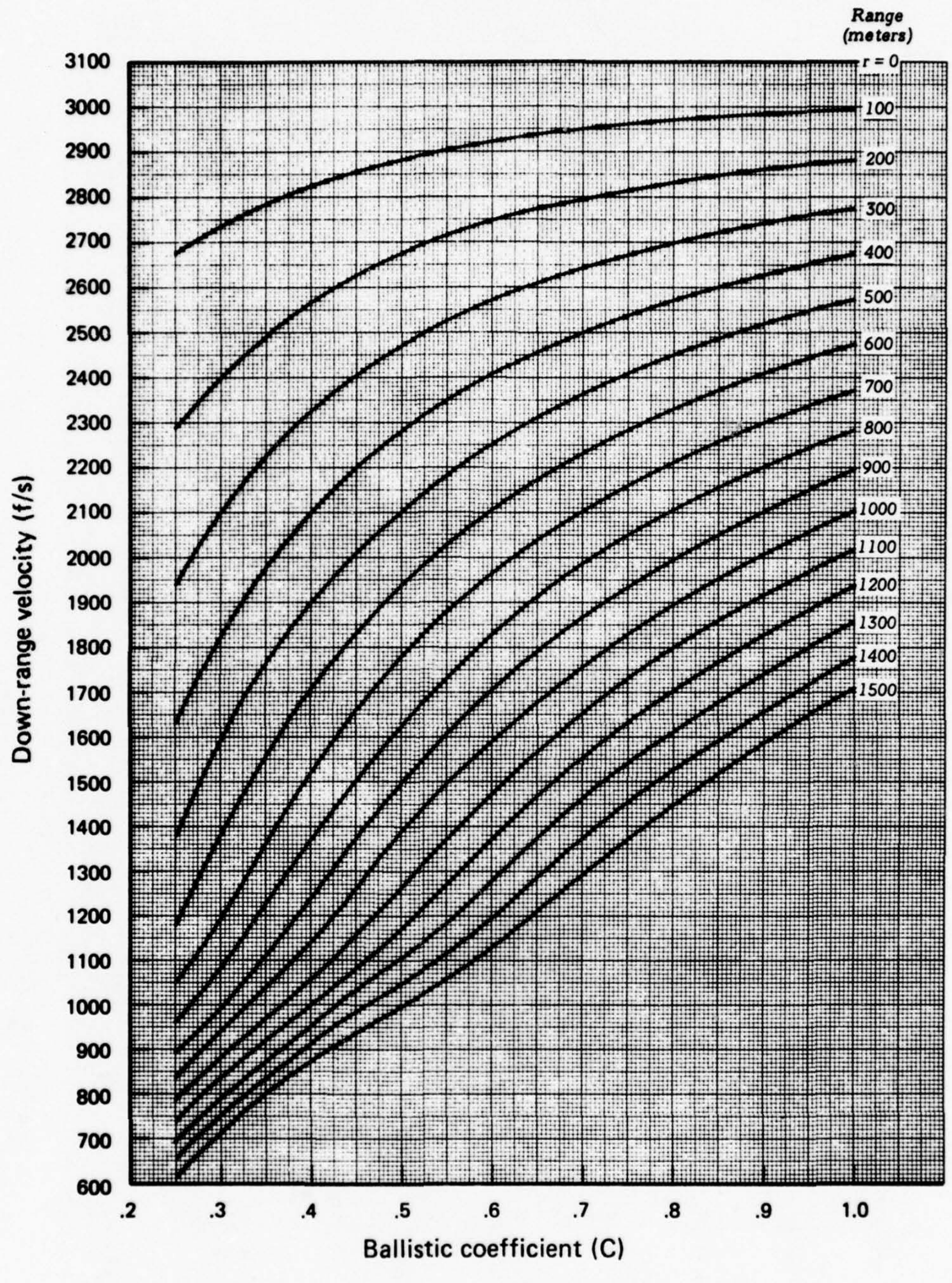

FIG. D-1.7: v AS A FUNCTION OF C FOR GIVEN r, v_o = 3100 f/s

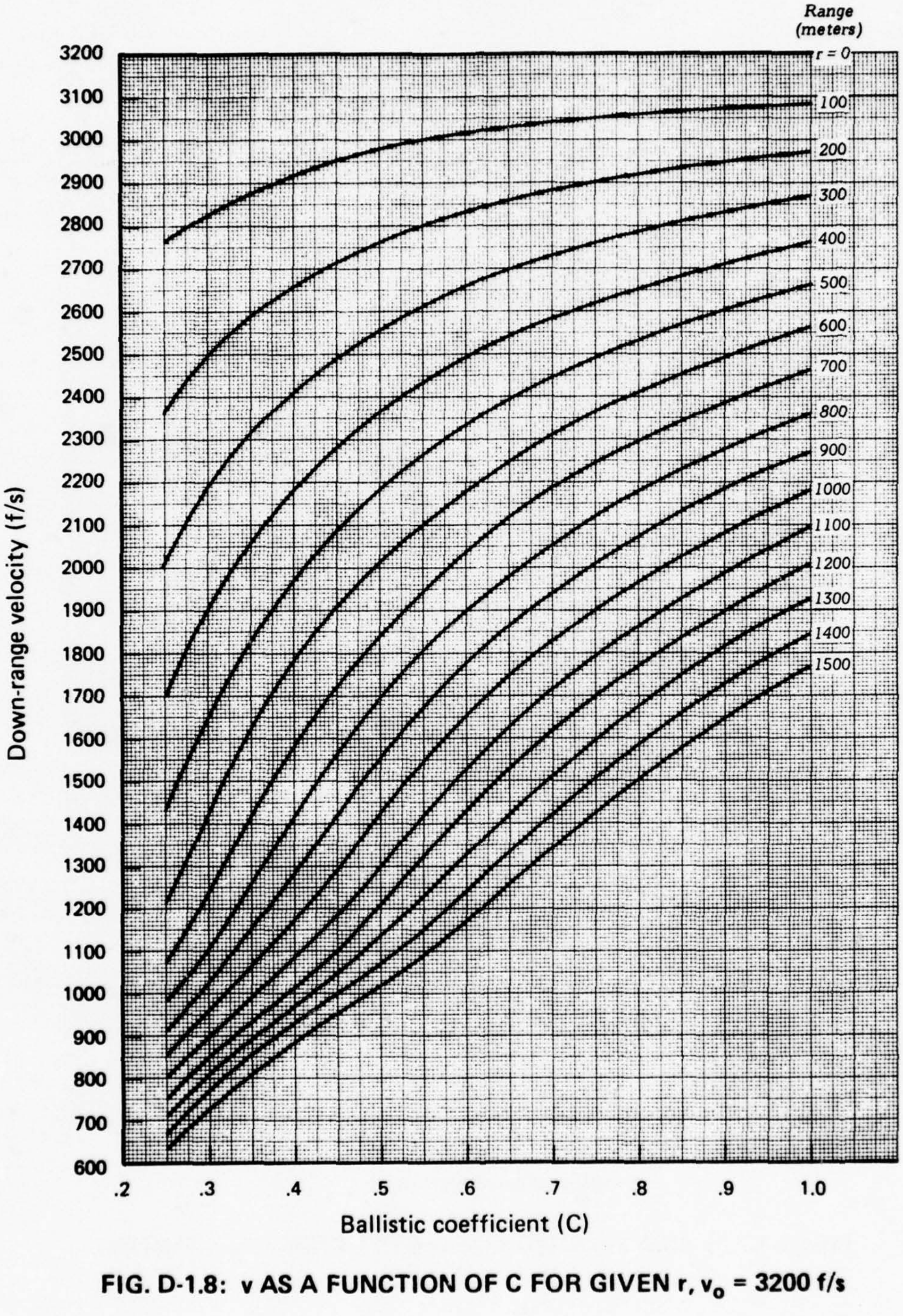

FIG. D-1.8: v AS A FUNCTION OF C FOR GIVEN r, v_o = 3200 f/s

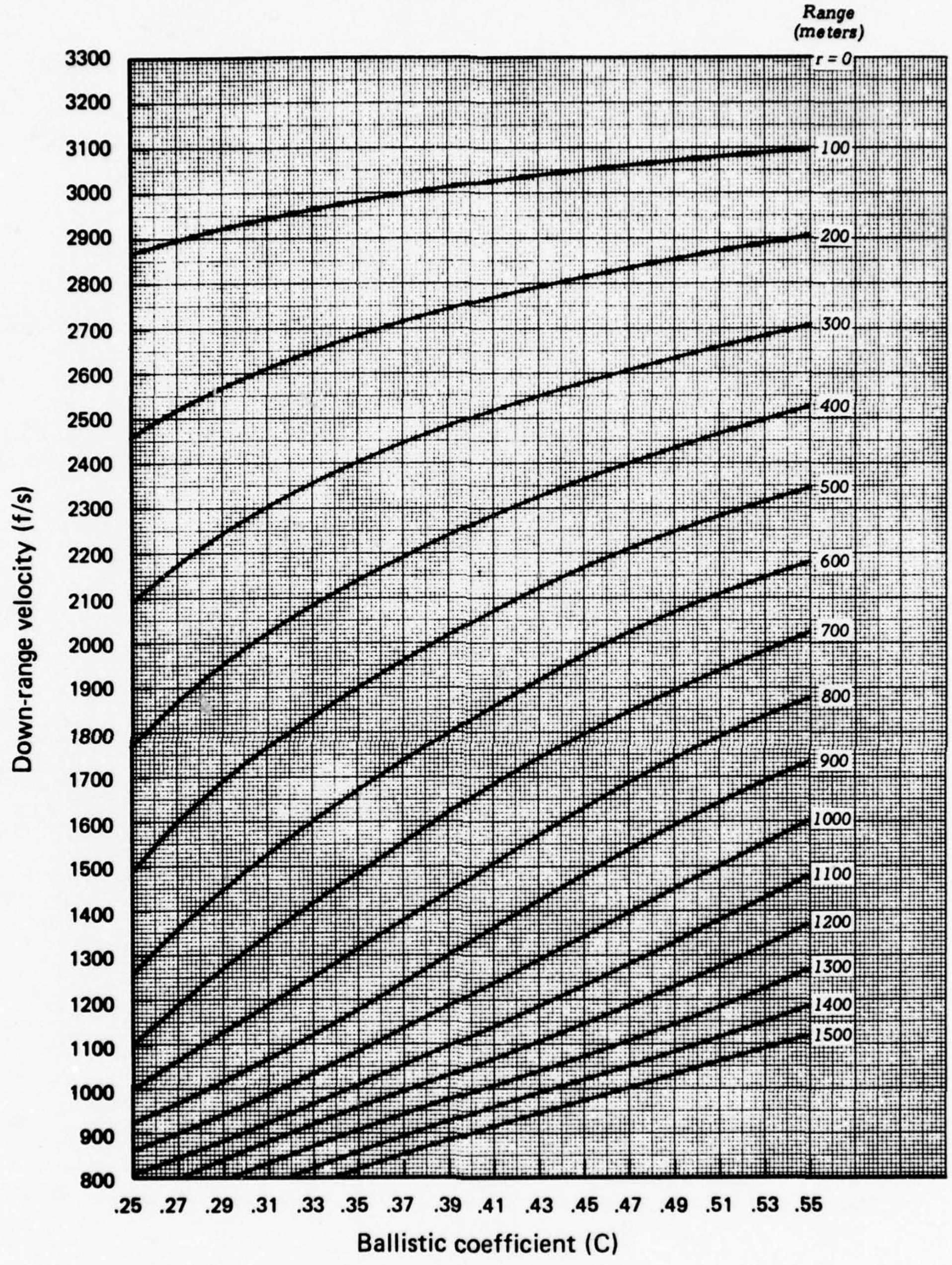

FIG. D-1.9: v AS A FUNCTION OF C FOR GIVEN r, v_o = 3300 f/s

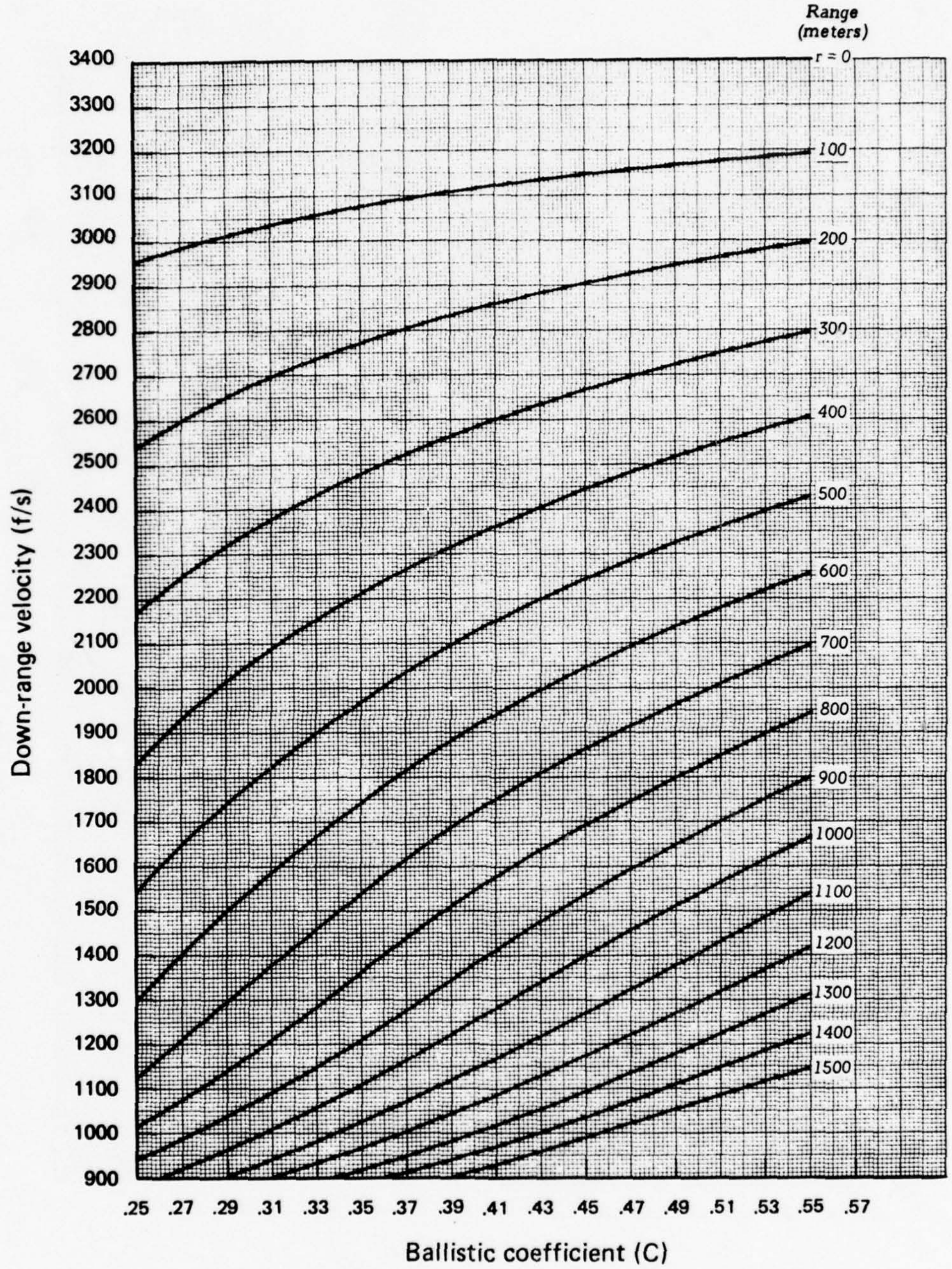

FIG. D-1.10: v AS A FUNCTION OF C FOR GIVEN r, v_o = 3400 f/s

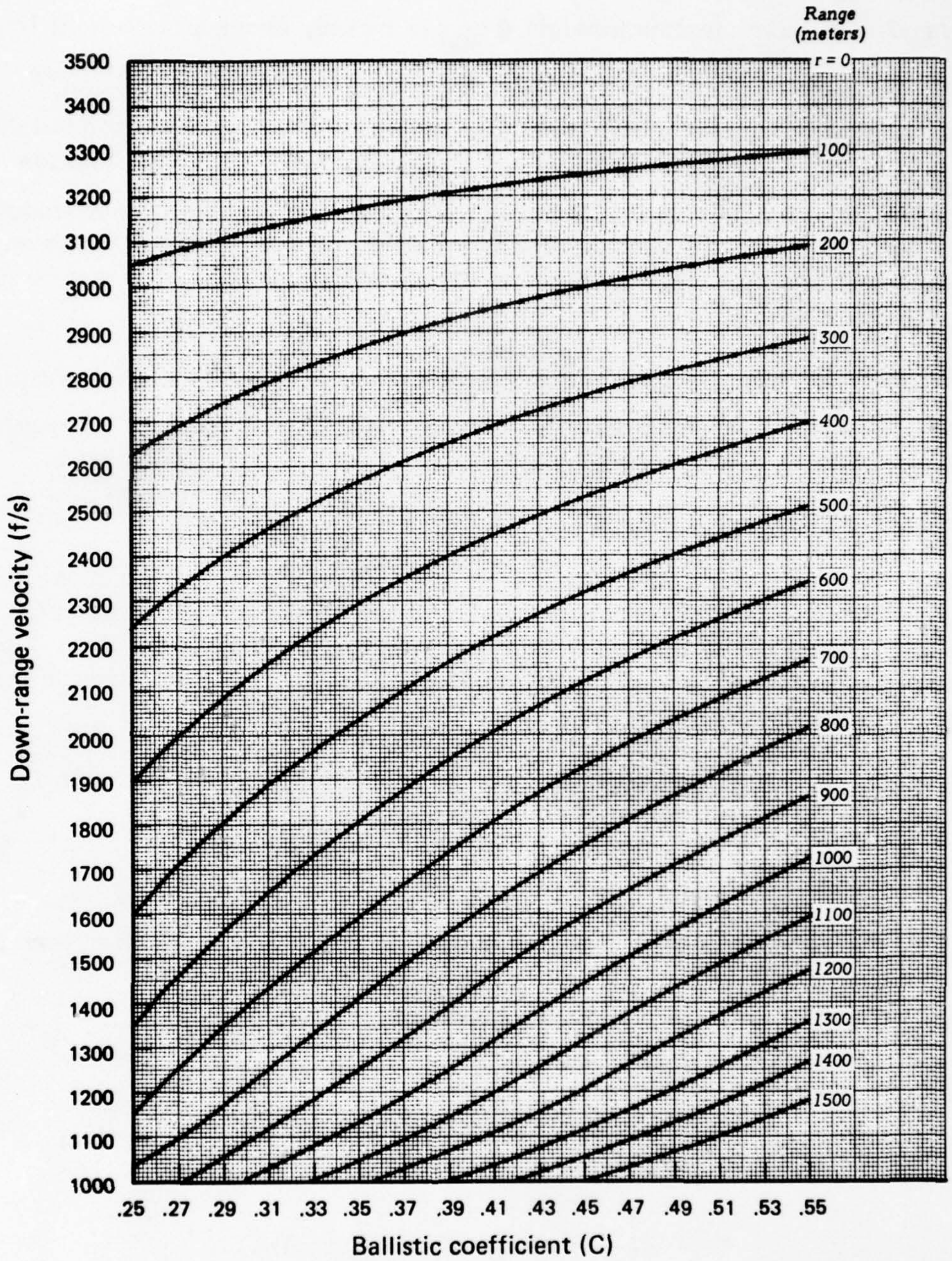

FIG. D-1.11: v AS A FUNCTION OF C FOR GIVEN r, v_o = 3500 f/s

Figure D-2 shows the maximum height (H_{300} in inches, above a horizontal line of sight) attained by a bullet aimed to strike a target at r = 300 meters. H_{300} is plotted as a function of ballistic coefficient C, each curve having an associated muzzle velocity v_o from the set 2,500(100)3,500. It should be noted H_{300} is a function of ballistic coefficient, muzzle velocity, and range (r = 300 meters) but is independent of caliber. It must, however, be kept in mind that caliber places practical limits on the ballistic coefficient and that large ballistic coefficients (say above 0.5) cannot be obtained with small-caliber rounds.

Tables D-1, D-2, and D-3 give estimates of H_{300} and other associated parameters for peak chamber pressures ($\hat{p}$) equal to 52,000, 50,000, and 48,000 psi, respectively (H_{300} in inches and $\hat{p}$ in pounds per square inch).

Drop

A second measure of trajectory flatness is drop. In the standard technical sense, drop is the vertical distance of the trajectory below the line of departure when the line of departure is horizontal. For purposes of this paper, we shall use the term "drop" in this standard technical sense, which is considerably more prevalent than the meaning given in some handbooks on reloading.

The drop D_r in inches (to range r) can be calculated from the formula

$$D_r = \left(T_r\right)^2 \; f\left(\frac{v_r}{v_o}\right) \quad . \qquad \text{(D-4)}$$

Values of $\left(T_r\right)^2$ from formula D-2, for a range of 300 meters, are shown in figure D-3 for

$$.35 \le C \le .85 \quad \text{and}$$

$$v_o = 2,500(100)3,500.$$

$f\left(\frac{v_r}{v_o}\right)$ is given in figure D-4 for

$$.32 \le \frac{v_r}{v_o} \le 1 \quad .$$

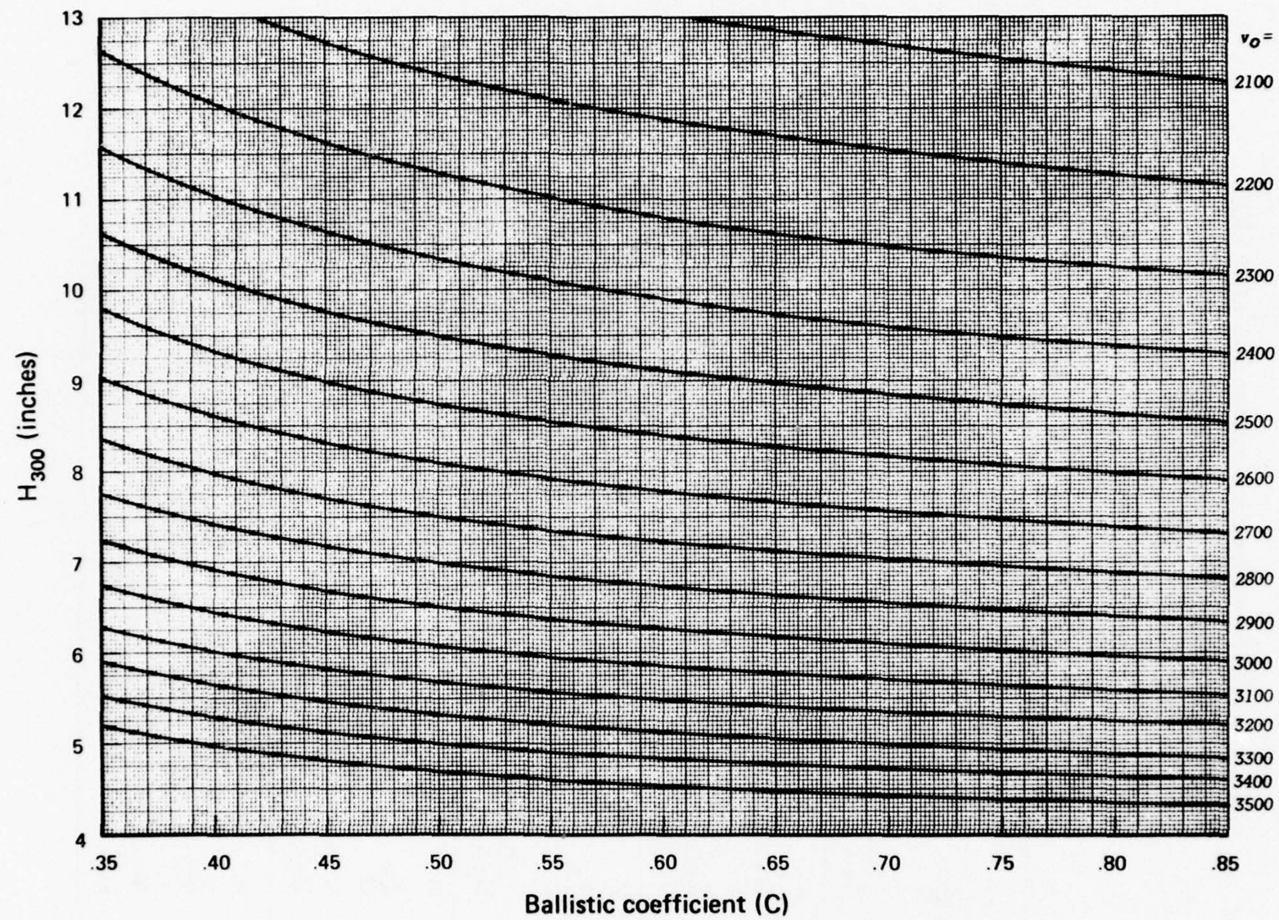

FIG. D-2: H_{300} AS A FUNCTION OF C FOR GIVEN v_o

TABLE D-1

HEIGHT OF TRAJECTORY OVER 300 METERS
(52,000 psi PEAK CHAMBER PRESSURE)

	d =	.224	.243	.264	.284	.308
TH	w	84.40	107.7	138.2	172.0	219.4
	v_o	3031	2910	2792	2692	2584
	$\bar{C}$	.455	.493	.536	.577	.625
	v_{300}	2357	2301	2249	2192	2134
	H_{300}	6.52	6.97	7.42	7.90	8.43
M/H	w	75.56	96.47	123.7	154.0	196.4
	v_o	3203	3075	2950	2845	2731
	$\bar{C}$	.407	.442	.480	.516	.560
	v_{300}	2430	2377	2320	2270	2212
	H_{300}	5.98	6.38	6.80	7.21	7.71
TM	w	66.73	85.19	109.2	136.0	173.5
	v_o	3408	3272	3140	3027	2906
	$\bar{C}$	.360	.390	.424	.456	.495
	v_{300}	2510	2458	2405	2358	2300
	H_{300}	5.45	5.78	6.18	6.54	6.98
M/L	w	57.90	73.92	94.78	118.0	150.5
	v_o	3659	3513	3371	3250	3120
	$\bar{C}$	.312	.339	.368	.396	.429
	v_{300}	2592	2547	2496	2462	2396
	H_{300}	4.91	5.21	5.53	5.85	6.24
TL	w	49.07	62.64	80.33	100.0	127.6
	v_o	3975	3816	3661	3530	3389
	$\bar{C}$	.264	.287	.312	.335	.364
	v_{300}	2672	2636	2593	2550	2503
	H_{300}	4.38	4.63	4.91	5.16	5.49

TABLE D-2

HEIGHT OF TRAJECTORY OVER 300 METERS
(50, 000 psi PEAK CHAMBER PRESSURE)

	d =	.224	.243	.264	.284	.308
TH	w	84.40	107.7	138.2	172.0	219.4
	v_o	2804	2692	2582	2490	2391
	$\bar{C}$	.455	.493	.536	.577	.625
	v_{300}	2164	2114	2062	2018	1965
	H_{300}	7.66	8.18	8.72	9.26	9.90
M/H	w	75.56	96.47	123.7	154.0	196.4
	v_o	2962	2844	2729	2631	2527
	$\bar{C}$	.407	.442	.480	.516	.560
	v_{300}	2227	2180	2131	2086	2036
	H_{300}	7.06	7.50	8.00	8.48	9.05
TM	w	66.73	85.19	109.2	136.0	173.5
	v_o	3153	3027	2904	2800	2689
	$\bar{C}$	.360	.390	.424	.456	.495
	v_{300}	2298	2252	2205	2162	2114
	H_{300}	6.44	6.83	7.28	7.68	8.20
M/L	w	57.90	73.92	94.78	118.0	150.5
	v_o	3385	3250	3118	3006	2887
	$\bar{C}$	.312	.339	.368	.396	.429
	v_{300}	2367	2330	2286	2245	2199
	H_{300}	5.81	6.16	6.54	6.90	7.35
TL	w	49.07	62.64	80.33	100.0	127.6
	v_o	3676	3530	3386	3265	3135
	$\bar{C}$	.264	.287	.312	.335	.364
	v_{300}	2434	2405	2368	2332	2291
	H_{300}	5.20	5.49	5.81	6.12	6.49

TABLE D-3

HEIGHT OF TRAJECTORY OVER 300 METERS
(48,000 psi PEAK CHAMBER PRESSURE)

	d =	.224	.243	.264	.284	.308
TH	w	84.40	107.7	138.2	172.0	219.4
	v_o	2578	2475	2375	2290	2198
	$\bar{C}$	.455	.493	.536	.577	.625
	v_{300}	1976	1931	1886	1840	1797
	H_{300}	9.14	9.73	10.39	11.04	11.80
M/H	w	75.56	96.47	123.7	154.0	196.4
	v_o	2725	2616	2510	2420	2323
	$\bar{C}$	.407	.442	.480	.516	.560
	v_{300}	2031	1991	1947	1907	1861
	H_{300}	8.42	8.96	9.52	10.10	10.78
TM	w	66.73	85.19	109.2	136.0	173.5
	v_o	2899	2784	2671	2575	2472
	$\bar{C}$	.360	.390	.424	.456	.495
	v_{300}	2091	2049	2012	1974	1931
	H_{300}	7.69	8.27	8.67	9.14	9.74
M/L	w	57.90	73.92	94.78	118.0	150.5
	v_o	3113	2989	2867	2764	2654
	$\bar{C}$	.312	.339	.368	.396	.429
	v_{300}	2150	2118	2080	2046	2005
	H_{300}	6.97	7.36	7.81	8.25	8.76
TL	w	49.07	62.64	80.23	100.0	127.6
	v_o	3381	3246	3115	3003	2883
	$\bar{C}$	.264	.287	.312	.335	.364
	v_{300}	2204	2180	2151	2120	2086
	H_{300}	6.26	6.59	6.96	7.32	7.76

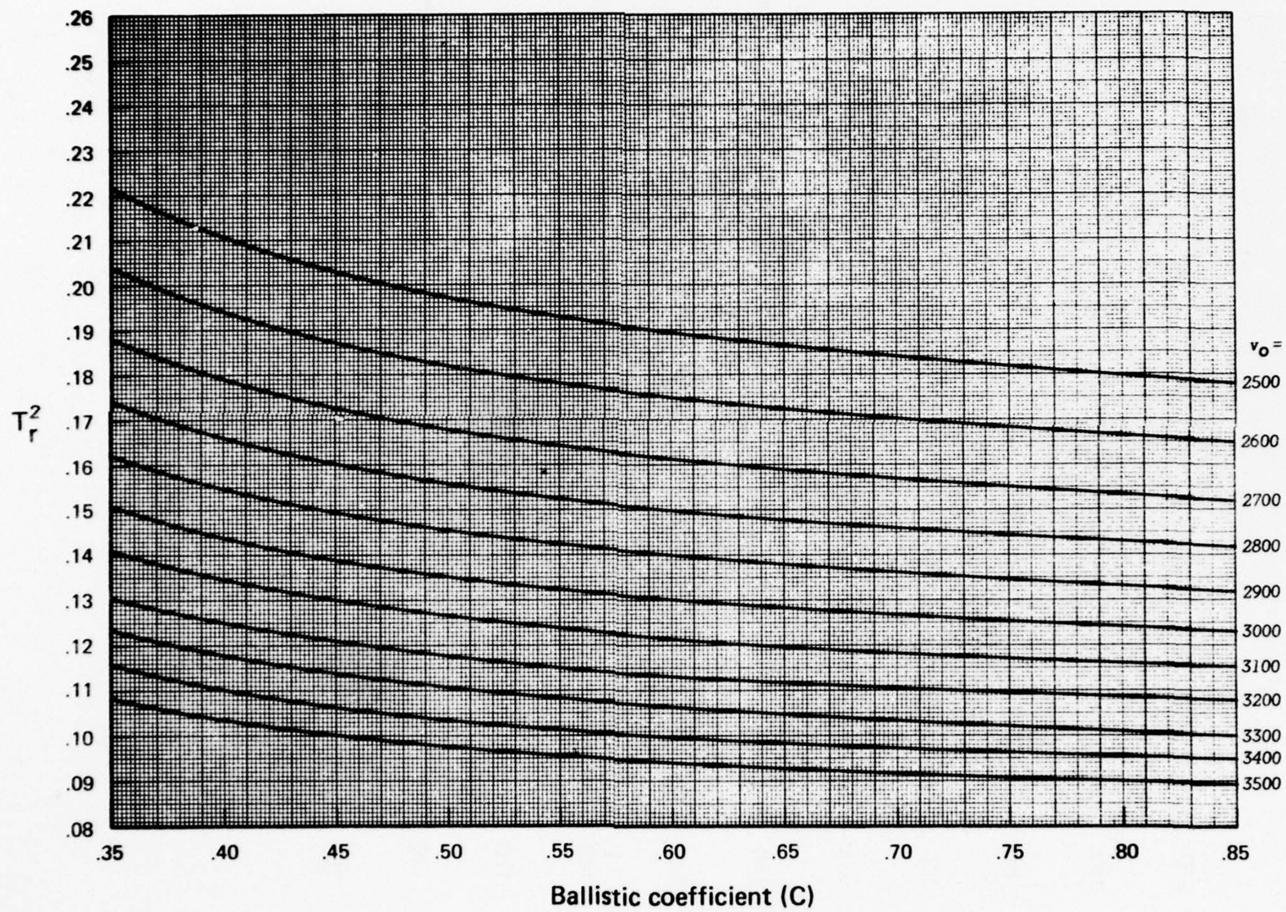

FIG. D-3: T_r^2 FOR RANGE r = 300 METERS AS A FUNCTION OF C FOR GIVEN v_o

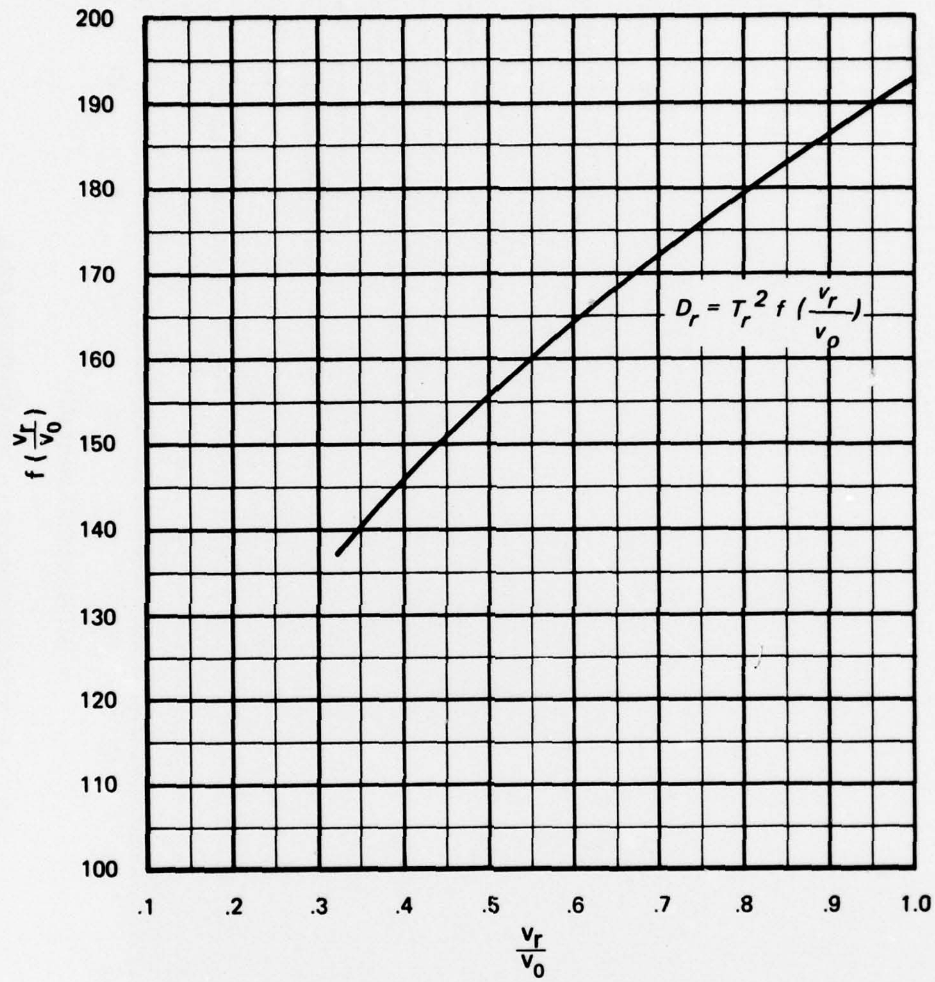

FIG. D-4: CURVE FOR CALCULATION OF DROP D

As an example, we will calculate H_r and D_r for $C = .59$, $v_o = 2{,}900$ fps, and $r = 300$ meters. From figure D-2, $H_{300} \simeq 6.75$ inches. From figure D-3, $T_r^2 = .140$, and from figure D-1.5, $v_r \simeq 2{,}380$ fps. Therefore, $\frac{v_r}{v_o} \simeq .821$ and, from figure D-4, $f(.821) = 181$. Hence, $D_r = (.140)(181) \simeq 25.3$ inches.

The measure H, rather than D, is of primary concern for the purposes of this paper. The data of the above example shows that if the rifle is zeroed at 300 meters, it is possible to fire at a target at any range closer than 300 meters without the bullet being more than 6.75 inches above the point of aim. Thus, for "large targets" the trajectory is sufficiently flat to 300 meters to permit firing without sight adjustment or hold under (aiming below the intended point of impact).

In practice, however, rifles are usually zeroed at a range which is less than the maximum range of interest, thus decreasing the trajectory height at ranges less than the zeroing range, and accepting some drop below the point of aim, at ranges from the zeroing range to the maximum range of interest.

Wind Deflection

The presence of wind during firing deflects the bullet's flight. While the expression "cross wind" is frequently encountered, any (non-zero) wind component, perpendicular to the trajectory path of the bullet, will cause deflection.

A common formula for wind deflection over the range r can be expressed as:

$$D_w = 17.60\,\bar{V}_w \left[T_r - \frac{r}{v_o} \right] , \qquad \text{(D-5)}$$

where

D_w = deflection measured in inches at the target

$\bar{V}_w$ = the average wind velocity component perpendicular to the line of flight, measured in miles per hour,

T_r = the time of flight to range r given by formula D-2, measured in seconds,

r = the range measured in feet,[1] and

v_o = the muzzle velocity measured in feet per second.

[1]Note that while r is usually in units of meters, feet are frequently specified by formulas relating to Ingalls Tables.

Since the force of the wind component (F_w) perpendicular to the bullets path is proportional to the bullet area presented (A_p), it appears that for constant wind velocity component $\bar{V}_w$,

$$F_w \propto A_p \quad .$$

For homologous bullets $A_p \propto d^2$ and $w \propto d^3$. Hence, from Newton's second law,

$$F_w \propto A_p \propto d^2 \propto w\, a \propto d^3 a \quad ,$$

where a is the instantaneous sidewise acceleration imparted to the bullet by the force of the wind.

Hence,

$$a \propto \frac{1}{d} \quad .$$

Thus, the instantaneous sidewise velocity of the bullet caused by the force of the wind satisfies the relation

$$v_w \propto \frac{t}{d} \quad ,$$

where t is the time since the bullet left the muzzle.

The sidewise displacement (wind deflection) of the bullet due to wind over range r, i.e., D_w satisfies the relations

$$D_w \propto \int_0^{T_r} \frac{t}{d} \, dt$$

$$D_w \propto \frac{T_r^2}{d} \quad .$$

Assuming this rather heuristic argument has first approximation validity, the wind deflection for homologous bullets to range r is directly proportional to the square of the time of flight and inversely proportional to caliber.

Three-Halves Retardation Law Approximations

Values of the trajectory characteristics discussed above can be approximated using the 3/2 power law for retardation. Retardation is a negative acceleration and the three-halves retardation law will yield rough approximations over the velocity range 2,000 $< v <$ 3,500 feet per second (reference D-1, page 559).

Expressed as a differential equation, the 3/2 law is:

$$\frac{dv}{dt} \simeq \frac{-A_1}{C} v^{3/2} \quad , \qquad \text{(D-6)}$$

where A_1 is a proportionality constant, C is the ballistic coefficient, v is velocity, and t is time.

Denoting the range by r, since

$$\frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \frac{dv}{dr} v \quad ,$$

D-6 can be written as:

$$v \frac{dv}{dr} \simeq \frac{-A_1}{C} v^{3/2} \quad . \qquad \text{(D-7)}$$

Integrating and solving for r yields,

$$\int_{v_o}^{v_r} v^{-\frac{1}{2}} dv \simeq \frac{-A_1}{C} \int_{o}^{r} dr \quad ,$$

$$r \simeq \frac{C}{A_2} \left[\sqrt{v_o} - \sqrt{v_r} \right] \quad , \qquad \text{(D-8)}$$

where v_o is the muzzle velocity, v_r is the velocity at range r (in feet), and $2A_2 = A_1$.

Formula D-8 can be used to estimate the range r (in feet) at which the velocity equals v_r if C and v_o are known and A_2 is determined.

Returning to D-6, we can also write

$$dv \simeq \frac{-A_1}{C} v^{3/2} \, dt \quad . \tag{D-9}$$

Integrating and solving for t_r, which denotes the time of flight to range r, we have

$$\int_{v_o}^{v_r} v^{-3/2} \, dv \simeq \frac{-A_1}{C} \int_{o}^{t_r} dt \quad ,$$

$$t_r \simeq \frac{C}{A_2} \left[\frac{1}{\sqrt{v_r}} - \frac{1}{\sqrt{v_o}} \right] , \tag{D-10}$$

where $2A_2 = A_1$ and r is in feet.

Solving D-8 for $\sqrt{v_r}$ gives

$$\sqrt{v_r} \simeq \sqrt{v_o} - \frac{A_2}{C} r \quad , \tag{D-11}$$

and substituting into D-10 yields

$$t_r \simeq \frac{C r}{C v_o - A_2 r \sqrt{v_o}} \quad , \tag{D-12}$$

where r is in feet and t_r is an estimate for T_r given by D-2. It should be noted that formula D-12 does not require v_r to be known.

To estimate A_2, the numerical values from Problem 5 (reference D-1, page 586) were substituted into D-8 and D-12 and the resulting values of A_2 averaged to obtain

$$A_2 \simeq 0.0028 \quad .$$

Values of t_r with $A_2 = 0.0028$ can be used to estimate T_r, H_r, and D_w. Following is a comparison of values T_r, H_r, and D_w when calculated from formulas D-2, D-3, and D-5 versus being estimated, using t_r as the estimate of T_r. The input parameters are: $r = 1640.5$ feet, $v_o = 2497$ feet per second, $C = .574$, and $\overline{V}_w = 5$ miles per hour.

$$T_r = 0.574\ [2.478 - 1.104], \text{ from formula D-2 and}$$

$$T_r = 0.789 \text{ seconds,}$$

where $v_r = 1,742$ feet per second; $T(v_r)$ and $T(v_o)$ must be looked up in Ingalls Tables.

$$t_r = \frac{(0.574)\,(1,640.5)}{0.574(2,497) - 0.0028(1,640.5)\ \sqrt{2,497}} \quad ,$$

$$t_r = 0.782 \text{ seconds,}$$

$$H_r = 48(T_r^2) = 29.9 \text{ inches,}$$

$$H_r = 48(t_r^2) = 29.4 \text{ inches,}$$

$$D_w(T_r) = 11.6 \text{ inches, and}$$

$$D_w(t_r) = 11.0 \text{ inches.}$$

KINETIC ENERGY

By definition, kinetic energy (E) is:

$$E = \tfrac{1}{2}\, m\, v^2 \quad ,$$

where m is the mass and v is the velocity in compatible units. For our application to bullets, this formula can be written:

$$E = \frac{1}{2}\ \frac{w}{7000g}\ v^2 \quad , \tag{D-13}$$

where w is the weight of the bullet in grains, v is its velocity in feet per second, g is the acceleration due to gravity (32.16 feet per second), 7000 is the number of grains in a pound, and E is in foot-pounds.

If a bullet having ballistic coefficient C is fired with muzzle velocity v_o, the down-range velocity (v) can be obtained from figures D-1.1 through D-1.11. This velocity (v) and the bullet weight (w) can be substituted in formula D-13 to yield the down-range kinetic energy E. Just as the down-range velocity can be shown graphically without employing the bullet weight, the down-range kinetic energy can be shown graphically without use of the ballistic coefficient. Figure D-5 shows v as a function of w for given values of E. Table D-4 gives values of J, $\bar{C}$, v_r, and E_r associated with table A-7.

EFFECTIVENESS

Two measures of effectiveness were considered for application in this paper. The first, developed at BRL and frequently referred to as the BRL three-halves incapacitation formula, is concerned with the probability of incapacitation given a hit. We decided not to use this formula, primarily because the experimental work upon which it is based did not employ small-arms bullets. The second measure considered is the kinetic energy (E). A difficulty with E as an effectiveness measure is the lack of agreement on the level of energy required to produce a given type of casualty. Also, while a bullet cannot transfer more energy than it possesses, the ability of the bullet to transfer its energy to the target is a crucial part of its effectiveness. The situation is further complicated by the fact that experience indicates that of two bullets with the same kinetic energy, the one with the larger mass tends to be more lethal. This belief implies that the kinetic energy formula gives too much weight to velocity as an effectiveness measure. Thus, the BRL three-halves incapacitation formula, which involves $v^{3/2}$, appears reasonable. It was decided to use kinetic energy (E) as the measure of effectiveness in the present paper. However, formulation of the energy required for a bullet to be lethal or effective still remains unresolved.

Data relating the energy (E) required for a projectile of a given weight (w) to defeat one side of the M-1 helmet (with liner) at 0 degrees obliquity was obtained from BRL. The energy required, as a function of bullet weight, is shown in figure D-6. It appears that the linear relationship shown in figure D-6 was obtained from data on three small-caliber bullets (M193, 68 Fed and 771WK) and three large-caliber bullets (M1943, Norma 139, and M80). Examination of bullet holes (in metal objects such as the M1 helmet, or sheets of metal) indicate that the hole is always somewhat larger than the bullet which produced it. Thus, one is led to visualize the hole as being opened up by the nose of the bullet so that the bearing surface of the bullet passes through without appreciable friction. It also seems reasonable to believe that the size of the hole that is opened is an important factor in determining the amount of energy required for penetration. A second factor affecting energy required to penetrate is the sharpness of

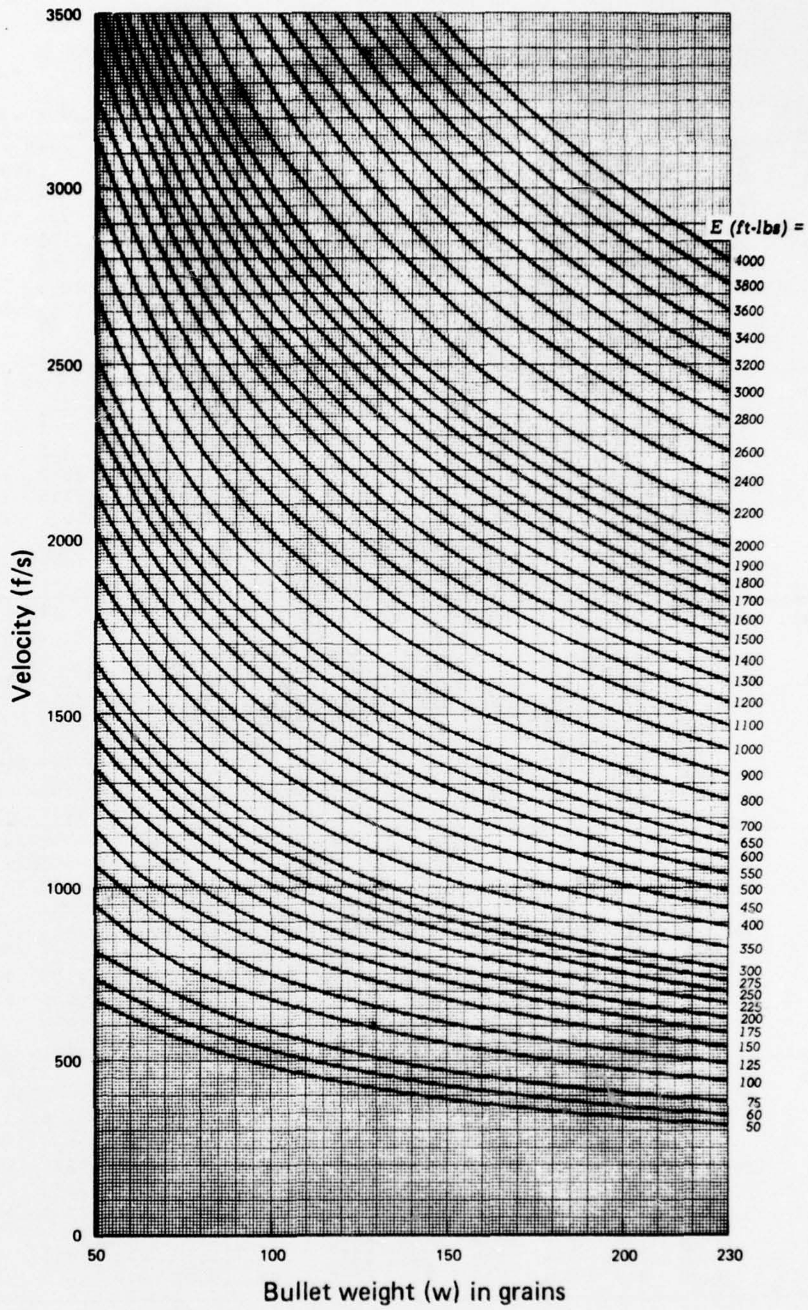

FIG. D-5: E, v, AND w RELATION

TABLE D-4

VELOCITY AND ENERGY AS A FUNCTION OF RANGE

Bull. Cat.			d = .224		d = .243		d = .264		d = .284		d = .308	
TH	w		84.40		107.7		138.2		172.0		219.4	
	c		.455		.493		.536		.577		.625	
	J		1.50		1.82		2.21		2.62		3.18	
			v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r
	r=	0	2804	1474	2692	1734	2582	2046	2490	2369	2391	2786
		100	2578	1246	2489	1482	2399	1767	2325	2065	2242	2449
		300	2164	878	2114	1069	2062	1305	2018	1556	1965	1882
		500	1799	607	1781	759	1760	951	1741	1158	1713	1430
		700	1488	415	1495	535	1498	689	1499	858	1492	1085
		1000	1146	246	1169	327	1191	435	1208	558	1222	728
M/H	w		75.56		96.47		123.7		154.0		196.4	
	c		.407		.442		.480		.516		.560	
	J		1.44		1.75		2.12		2.52		3.05	
			v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r
	r=	0	2962	1472	2044	1733	2729	2046	2631	2368	2527	2786
		100	2703	1226	2610	1460	2518	1742	2439	2035	2355	2417
		300	2227	833	2180	1019	2131	1248	2086	1488	2036	1809
		500	1814	552	1804	697	1788	878	1770	1072	1749	1335
		700	1467	361	1484	472	1494	613	1497	767	1499	980
		1000	1109	206	1136	277	1161	371	1181	477	1202	629
TM	w		66.73		85.19		109.2		136.0		173.5	
	c		.360		.390		.424		.456		.495	
	J		1.39		1.68		2.03		2.41		2.92	
			v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r
	r=	0	3153	1473	3027	1734	2904	2045	2808	2368	2689	2786
		100	2850	1204	2753	1434	2658	1714	2575	2003	2487	2383
		300	2298	783	2252	960	2205	1180	2162	1412	2114	1722
		500	1824	493	1819	626	1810	795	1798	977	1782	1224
		700	1435	305	1457	402	1477	529	1488	669	1497	864
		1000	1066	169	1094	227	1123	306	1147	397	1171	529
M/L	w		57.90		73.92		94.78		118.0		150.5	
	c		.312		.339		.368		.396		.429	
	J		1.34		1.61		1.94		2.30		2.70	
			v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r
	r=	0	3385	1474	3250	1734	3118	2047	3006	2368	2887	2786
		100	3023	1175	2924	1404	2824	1679	2737	1963	2644	2337
		300	2367	721	2330	891	2286	1100	2245	1321	2199	1616
		500	1816	424	1824	546	1823	700	1819	867	1808	1093
		700	1377	244	1414	328	1442	438	1462	561	1479	731
		1000	1012	132	1044	179	1074	243	1100	317	1127	425
TL	w		49.07		62.64		80.33		100.0		127.6	
	c		.264		.287		.312		.335		.364	
	J		1.28		1.54		1.85		2.19		2.64	
			v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r	v_r	E_r
	r=	0	3676	1473	3530	1734	3386	2046	3265	2368	3135	2785
		100	3230	1137	3128	1361	3024	1632	2934	1912	2836	2279
		300	2434	645	2405	805	2368	1001	2332	1208	2291	1488
		500	1779	345	1803	452	1816	589	1820	736	1823	942
		700	1286	180	1335	248	1377	338	1407	440	1438	586
		1000	947	98	980	134	1012	183	1039	240	1070	325

r in meters $\quad$ w in grains $\quad$ $\tilde{p} = 20,390$ lb/in²

v_r in fps $\quad$ E_r in ft lbs $\quad$ $\hat{p} \simeq 50,000$ lb/in²

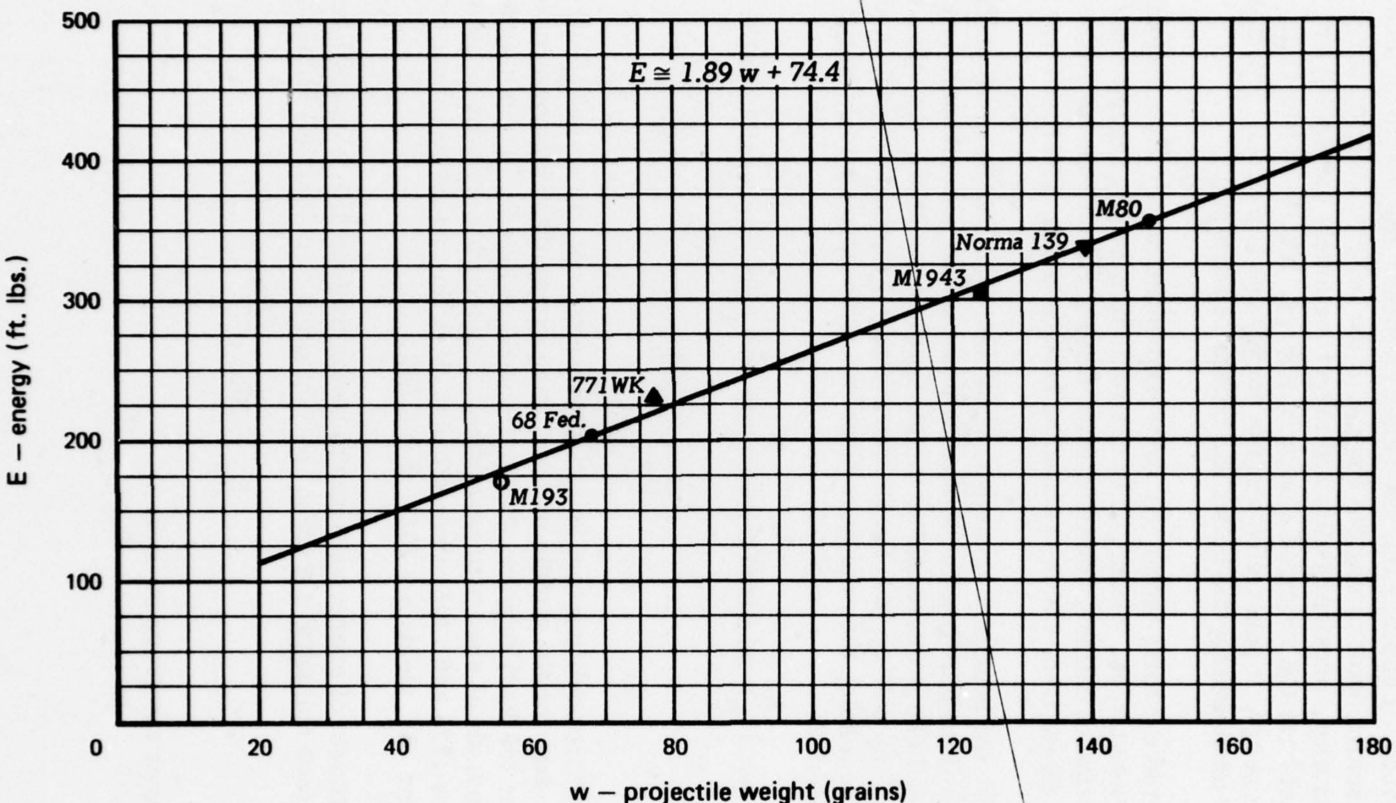

FIG. D-6: ENERGY vs. WEIGHT FOR PROJECTILE DEFEATING ONE SIDE OF M1 HELMET WITH LINER AT 0° OBLIQUITY – LEAD CORE

the bullet nose. Hence, it appears that the bullet diameter (d) and form factor (i) are two principal factors determining the amount of the bullet's kinetic energy used up during the penetration, provided bullet and target materials (hardness) are fixed. If the bullet does not have the required amount of energy, it fails to penetrate -- but, if it has sufficient energy, it penetrates and the energy used up during penetration is independent of how much energy the bullet had before penetration.

We have required the form factor (i) to be held constant (for all bullet calibers and weights) for comparisons. Hence, it is desirable to have the energy required to penetrate as a function of caliber, for fixed form factor, rather than as a function of bullet weight as in figure D-6, which is based on bullets having different form factors.

To continue with the heuristic argument, if the bullet (in penetration) produced only radial fracture lines plus bending along segments of the hole circumference, the energy required should be proportional to the diameter of the hole -- in other words, to the first power of the caliber d. On the other hand, if the bullet "ground out the material" to produce a hole, the energy required should be proportional to the second power of the diameter d.

Thus, the energy (E_p) required to penetrate should be expressible in the form

$$E_p = \alpha(d)^{\beta} \quad ,$$

where α is the proportionality factor and β is the power of d. α and β were estimated by using the data for the middle bullet of each group of three shown in figure D-6. The smaller caliber (68 Fed) requires 205 foot-pounds, while the larger (Norma 139) requires 335 foot-pounds of energy to penetrate. These energy requirements were then assigned to calibers .224 and .308, respectively. Substituting these values into the above expression for E_p yields $\alpha = 2060$, $\beta = 1.542$.

Hence, the estimate of energy required to penetrate (one side of the M1 helmet with liner at 0 degrees obliquity) expressed as a function of caliber (d) is taken as,

$$E_p = 2060\ d^{1.542} \quad . \qquad \text{(D-14)}$$

Figure D-7 shows the graph of E_p.

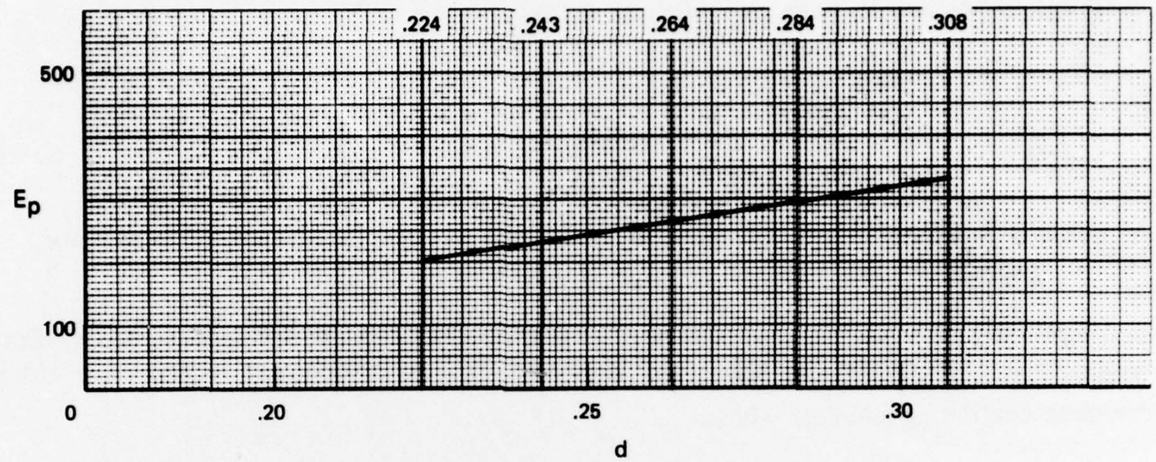

FIG. D-7: E_p AS A FUNCTION OF d

Hatcher and others have concluded that about 100 foot-pounds of energy are required to produce a wound. Hence, one formula for estimating the energy (E^*) required for a bullet to be effective is to use:

$$E_1^* = 100 + E_p$$

$$E_1^* + 100 + 2060\, d^{1.542} \qquad \text{(D-15)}$$

While 100 foot-pounds of energy may be sufficient to produce a wound, many authorities in police work feel that the 38 Special is marginal when it comes to putting the criminal out of the fight and is not satisfactory for use against vehicles. The 38 Special has been the police standard sidearm for many years, but recently the 357 magnum has been gaining favor with some police departments. Depending on the particular loading, the 38 Special has about 300 foot-pounds of energy at 25 yards, while the 357 magnum has on the order of 700 foot-pounds of energy.

It seems obvious that before one decides on energy requirements for effectiveness, the target must be considered. Energy at the 300 foot-pounds level may be adequate against unprotected personnel but a 700 foot-pound level may be required for effectiveness against vehicles, such as jeeps and trucks.

Thus, we propose:

$$E_2^* = 300 \text{ foot-pounds} \quad , \qquad \text{(D-16)}$$

as the (expression for) energy required by a bullet to have "knock-down" capability against unprotected personnel.

Another approach would be to require the bullet to have energy at least equal to the larger of the two values (E_1^*, E_2^*). This gives E_3^* as the energy required for effectiveness against personnel, where

$$E_3^* = \text{Maximum } (E_1^*, E_2^*)^{1} \quad . \qquad \text{(D-17)}$$

The implications of such choices in defining the amount of energy required for effectiveness are so pronounced that it may be impossible to reach agreement on a quantitative definition of maximum effective range for small arms. An approach which appears much more useful (than the MER concept) is to employ various individual measures of effectiveness such as the above E_1, E_2, E_3, sand bag penetration, pine board penetration, or the BRL three-halves incapacitation formula.

[1]For calibers considered in this paper, E_1^* is always greater than E_2^*.

REFERENCES

D-1 Hatcher, Julian, "Hatcher's Notebook", The Stockpole Co., 1966